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THE TRANSMISSION DYNAMICS OF THE HUMAN IMMUNODEFICIENCY VIRUS TYPE 1 IN THE MALE HOMOSEXUAL COMMUNITY IN THE UNITED KINGDOM: THE INFLUENCE OF CHANGES IN SEXUAL BEHAVIOUR

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This paper examines the transmission dynamics of human immune deficiency virus type 1 (HIV-1) in the male homosexual population in the U.K. via numerical studies employing a mathematical model representing the principal epidemiological process. The model is based on an assumption of proportionate mixing between different sexual-activity classes (defined by the rate of sexual partner change per unit of time) and incorporates heterogeneity in sexual activity, distributed infection and incubation periods and the recruitment of susceptibles to the sexually active population. The sensitivity of model predictions to various assumptions and parameter assignments is examined. Numerical studies of model behaviour focus on the influence of changes in the magnitudes of the transmission parameters, associated with three periods of infectiousness during the incubation period of acquired immune deficiency syndrome (AIDS), on the magnitude and duration of the epidemic and on the level of the endemic equilibrium state. Predicted temporal trends in the incidence of AIDS are shown to be particularly sensitive to changes in the intensities and durations of the stages of infectiousness. Most of the paper addresses the influence of changes in sexual behaviour on the magnitude and duration of the epidemic. Numerical simulations show that the manner in which behavioural changes occur and who is influenced by such changes (i.e. infecteds or susceptibles, the sexually active population or new recruits to this population) have a major impact on the future timecourse of the epidemic. The greatest reduction in the incidence of AIDS over the coming decades is induced by changes in the rate of sexual-partner change among the sexually active population, particularly those currently infected. The time periods at which changes in behaviour occur, in relation to the starting point of the epidemic (assumed to be 1979), are also of particular significance to the future pattern of the incidence of disease and infection. Changes in behaviour early on in the timecourse of the epidemic have a much greater impact than equivalent changes at latter time points. On the basis of limited data on the pattern of change in sexual behaviour among the male homosexual community in the U.K., numerical studies of model behaviour tentatively suggest that the epidemic is at, or near to, a period of peak incidence of the disease AIDS. Analyses suggest that, following the peak in incidence, there will be a period of slow decline over many decades provided recent changes in behaviour are maintained in the coming years. The difficulties surrounding model formulation and the prediction of future trends are stressed. Many uncertainties remain concerning parameter assignments and emphasis is placed on the need for better data on patterns of sexual behaviour and changes therein, infectiousness throughout the long and variable incubation period of AIDS and the number of individuals currently infected with HIV-1 in the U.K.

1. Introduction

Mathematical models of infection and disease may serve as illuminating caricatures of observed pattern and current hypotheses, as analytic tools for the estimation of epidemiological parameters, as guides to the information needed to improve understanding and as a template for planning health-care provision and programmes of control. Over the past few years increasing attention has been focused on the application of statistical and mathematical methods to the study of the epidemiology of the acquired immune deficiency syndrome (AIDS) and the transmission dynamics of the aetiological agent of the disease, the human immunodeficiency virus type 1 (HIV-1) (Anderson 1988*a*).

This research falls into five broad areas. The first concerns short-term prediction of the number of cases of AIDS by the use of simple mathematical functions such as the exponential or the logistic (Norman 1985; McEvoy & Tillet 1985). The parameters of the models may be estimated by fitting the functions to the recorded longitudinal patterns of AIDS incidence, and various refinements may be included such as a distributed lag between diagnosis and reporting (Cox & Medley 1989). The second area is the use of models to estimate particular epidemiological parameters such as the rate of infection per head, the probability of transmission per sexual contact and summary statistics of the distribution of the incubation period (Lui *et al.* 1986; Medley *et al.* 1987; Grant & Wiley 1987; Winkelstein *et al.* 1987; May & Anderson 1987; Anderson 1988*b*). The fourth area is the development of models to describe the dynamic interaction between HIV-1 and the human immune system within individual patients. To date this area has received little attention but it is an important topic for future research (Cooper 1986; Anderson 1988*a*). The fifth and final area is the subject of this paper and concerns the development of models to describe the transmission dynamics of the virus within and between defined at-risk groups. Most past studies have centred on the principal at-risk group, namely, male homosexuals, in developed countries (Anderson *et al.* 1986; Dietz & Hadeler 1988; Hymen & Stanley 1988; May & Anderson 1988). This group currently accounts for more than 70% of reported cases in countries such as the U.S.A. and the U.K.

In this paper we examine the transmission dynamics of HIV-1 in the male homosexual population in the U.K. via the development and analysis of a simple model that incorporates distributed infectious and incubation periods, heterogeneity in sexual activity (defined as the number of different sexual partners per unit of time) and continual recruitment of susceptible individuals to the population. The model is based on published research (see Anderson *et al.* 1986, 1987; Anderson & May 1986; May & Anderson 1987; Blythe & Anderson 1988*a*, *b*, *c*) and the paper examines the sensitivity of model predictions to various assumptions and parameter assignments via extensive numerical studies. The primary focus of these analyses is an assessment of the influence of changes in sexual behaviour on the likely course of the epidemic in the coming decades.

The paper is organized as follows. The first section provides a very brief overview of recent progress towards an understanding of the major epidemiological processes that determine the pattern of the epidemic in the male homosexual community. The second section outlines the structure of the model and the various assumptions made in its formulation. Particular attention is given to the formulation of the transmission term and the treatment of distributed infectious and incubation periods. The third and longest section summarizes the results of extensive numerical investigations of the properties of the model by reference to temporal

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trends in the incidence of disease (AIDS) and the proportion of the community infected (seroprevalence) with HIV-1, stratified by sexual activity class (the rate of change in sexual partners). The final discussion section pays particular attention given to the influence of changes in sexual behaviour on the likely pattern of the epidemic in the U.K. over the coming decades.

2. Epidemiological patterns and parameters

The overall pattern of spread of HIV infection within a defined risk group depends on the magnitude of the basic reproductive rate, R_0, which measures the number of new cases of infection produced, on average, by an infected individual in the early stages of the epidemic when virtually all contacts are susceptible. The basic reproductive rate is a composite parameter formed from the product of the average probability β, that an infected individual will infect a susceptible sexual partner over the duration of their relationship, multiplied by the effective average number of new partners acquired per unit of time, c (within the specified risk-group), multiplied by the average duration of infectiousness, D:

$$R_0 = \beta c D. \tag{1}$$

There are, of course many problems associated with what exactly is meant by 'average' with respect to the three components in this definition. These problems are addressed in the following section on model formulation. The quantity R_0 simply helps to focus attention on what must be measured to gain a crude understanding of the pattern of spread of infection (Anderson *et al.* 1986; May & Anderson 1987, 1988).

Since the discovery of the aetiological agents of AIDS, much research has been directed towards the measurement of the components encapsulated in the definition of R_0. However, many uncertainties remain, largely as a result of the long and variable incubation period of the disease (many years) and as a consequence of public and professional sensitivities surrounding the study of sexual behaviour in human communities. A recent review of progress towards parameter assignments that indicates the problems and uncertainties that are emerging in such research is given by Anderson & May (1988). In this section we briefly summarize current understanding and indicate the major areas of uncertainty.

(*a*) *The proportion of infecteds who develop* AIDS

Prospective studies of homosexual men infected with HIV show that the rate of progression from infection to AIDS is slow. The rate of progression appears to vary with time from infection, being low in the first year and rising thereafter. Estimates of the fraction f who will go on to develop AIDS have increased with time. Rates of progression from seropositivity to AIDS derived from cohorts where the dates of seroconversion are not always known range from less than 1% to more than 12% per year with an average around 5–7%. Rates of progression to AIDS are much higher in perinatally infected young children as well as in elderly people, by comparison with sexually active adults. Current evidence suggests that between 30 and 75% of infected individuals will have progressed to AIDS within six years. Overall, it is likely that a very high fraction (perhaps greater than 90%) will ultimately progress to AIDS with the only uncertainty being the distribution of times to the development of disease (table 1). Therapy with azidothymidine is likely to influence this distribution because studies suggest that the drug

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TABLE 1. PROPORTION (p) OF HIV ANTIBODY POSITIVE INDIVIDUALS DEVELOPING AIDS

risk group	sample size	observation time (months)	p	reference
haemophiliacs	18	24	0.06	Simmonds *et al.* (1988)
haemophiliacs	40	36	0.13	Goedert *et al.* (1986)
haemophiliacs	92	48	0.11	Eyster *et al.* (1987)
haemophiliacs	84	54	0.12	Eyster *et al.* (1987)
haemophiliacs	77	?	0.13	Jason *et al.* (1988)
IV drug users	165	9	0.02	Des Jarlais *et al.* (1987)
IV drug users	72	14	0.06	Vaccher *et al.* (1988)
IV drug users	77	18	0.07	Zulaica *et al.* (1988)
IV drug users	50	24	0.32	Fernandez-Cruz *et al.* (1988)
IV drug users	181	24	0.12	Selwyn *et al.* (1988)
IV drug users	102	36	0.09	Crovari *et al.* (1988)
IV drug users	24	36	0.13	Goedert *et al.* (1986)
ex-IV drug users	61	36	0.03	Crovari *et al.* (1988)
PGL patients	88	15	0.08	Weller *et al.* (1985)
PGL patients	90	19	0.17	Weller *et al.* (1985)
PGL patients	101	20	0.60	Gottlieb *et al.* (1987)
PGL patients	100	24	0.13	Carne *et al.* (1987)
PGL patients	22	35	0.46	Harrer *et al.* (1988)
PGL patients	42	60	0.29	Weller *et al.* (1985)
PGL patients	200	61	0.08	Abrams *et al.* (1985)
PGL patients	200	74	0.38	Abrams *et al.* (1988)
PGL patients	75	72	0.38	Kaplan *et al.* (1988)
PGL & ARC patients	68	20	0.76	Gottlieb *et al.* (1987)
homosexuals	1835	15	0.03	Polk *et al.* (1987)
homosexuals	813	18	0.07	Gottlieb *et al.* (1987)
homosexuals	233	24	0.80	Antonen *et al.* (1987)
homosexuals	42	24	0.14	Detels *et al.* (1987)
homosexuals	306	30	0.10	de Wolf *et al.* (1988)
homosexuals	30	31	0.23	Gerstoft *et al.* (1987)
homosexuals	96	32.5	0.70	Schechter *et al.* (1988)
homosexuals	34	36	0.26	Pederson *et al.* (1987)
homosexuals	33	36	0.12	Weber *et al.* (1986)
homosexuals	86	36	0.22	Goedert *et al.* (1987)
homosexuals	44	36	0.34	Goedert *et al.* (1986)
homosexuals	288	44	0.17	Moss *et al.* (1988)
homosexuals	42	36	0.17	Goedert *et al.* (1986)
homosexuals	26	36	0.08	Goedert *et al.* (1986)
homosexuals	18	48	0.22	El-Sadr *et al.* (1987)
homosexuals	30	48	0.07	Titti *et al.* (1988)
homosexuals	246	50	0.15	Schechter *et al.* (1988)
homosexuals	115	53	0.28	Karlsson *et al.* (1988)
homosexuals	86	60	0.56	Biggar *et al.* (1988)
homosexuals	31	61	0.06	Jaffe (1985)
homosexuals	1100	72	0.04	Curran *et al.* (1985)
homosexuals	71	86	0.42	Hessol *et al.* (1988)
mothers	16	30	0.31	Scott *et al.* (1985)
children of HIV+ mothers	178	12	0.60	European Collaborative Study (1988)

suppresses viral replication in certain patients (Parks *et al.* 1988). Whether or not drug treatment will prevent the development of AIDS in some patients over very long time periods (i.e. decades) is uncertain at present.

(*b*) *The incubation period of* AIDS

A variety of parametric and non-parametric methods have been used to estimate summary statistics of the incubation periods of AIDS (defined as the time interval between infection and

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the diagnosis of symptoms of disease) (see Lui *et al.* 1986; Medley *et al.* 1987, 1988). A summary of published estimates of certain statistics (e.g. the mean or the median) is presented in table 2 for various risk groups.

TABLE 2. INCUBATION PERIOD

risk group	incubation/years mean	incubation/years median	sample size	reference
U.S.A. heterosexual prisoners, IV drug abusers	1.91	—	14	Hanrahan *et al.* (1984)
U.S.A. homosexuals	0.88	—	12	Auerbach *et al.* (1984)
U.S.A. TA adults over 13 years	4.5	—	83	Lui *et al.* (1986)
U.S.A. TA adults	15	—	144	Rees (1987)
U.K. TA cases	3.1	—	393	MRC/DHSS project (1987)
U.S.A. TA adults 60 years+	5.5	5.44	135	Medley *et al.* (1987)
U.S.A. TA persons below 5 and 59 years	8.23	7.97	126	Medley *et al.* (1987)
U.S.A. TA female cases	8.7	8.36	112	Medley *et al.* (1987)
U.S.A. TA male cases	5.62	5.5	185	Medley *et al.* (1987)
U.S.A. TA children under 12 years	1.97	1.9	36	Medley *et al.* (1987)
U.S.A. TA children under 12 years	2.03	—	40	Rogers *et al.* (1987)
U.S.A. TA children under 12 years	2.58	—	90	Rogers *et al.* (1988)
U.S.A. perinatally infected under 12 years	—	5.4	215	Auger *et al.* (1988)
U.S.A. children	—	0.5 to 5.5	190	Lawrence *et al.* (1988)
U.S.A. TA cases	7.29	7.63	440	Valleron *et al.* (1988)
U.S.A. TA cases				
(Weibull)	4.93	5.21	—	Valleron *et al.* (1988)
(Gamma)	14.11	11.38	—	Valleron *et al.* (1988)
U.S.A. TA adults over 12 years				
(Weibull)	7.59	7.32	512	Medley *et al.* (1988)
(Gamma)	24.07	20.84	—	Medley *et al.* (1988)
U.S.A. Haemophiliacs over 12 years				
(Weibull)	7.66	7.43	1401	Anderson, Medley (1988)
(Gamma)	14.33	12.61	—	Anderson, Medley (1988)

Parametric approaches to the estimation of the distribution of incubation periods in cohorts of individuals within a given risk group (i.e. male homosexuals, intravenous drug users, perinatally infected infants and haemophiliacs), for whom times of infection or seroconversion are known, are based on the development of simple mathematical models. These must take into account the growth in the number of infected individuals through time (i.e. exponential in the early stages of the epidemic), the proportion of infecteds who will ultimately develop AIDS and the form of the probability distribution of the incubation period. With respect to the last, the Weibull and γ-distributions have been used in many of the published studies (see Lui *et al.* 1986; Medley *et al.* 1987, 1988). In the context of the construction of models of the transmission dynamics of the virus, a parametric assumption concerning the form of the distribution is essential for predicting future patterns or trends. Published estimates of the means of Weibull and γ-distributions fitted to observed data (where only a given fraction of the distribution has been observed to date) have wide confidence limits (Kalbfleisch & Lawless 1988). However, for transfusion-associated cases and cases in male homosexuals as well as in intravenous (IV) drug users, estimates of the mean tend to lie in the range of 6–8 years (see table 2). This average appears to be somewhat less in children (less than 12 years) and elderly people (greater than 60 years). The disease in infants infected perinatally appears to have a very short incubation period with averages in the range of 1–3 years (Rogers *et al.* 1987).

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(c) *Infectious period*

The average period during which infected individuals are infectious to susceptible partners, over the long and variable incubation period of AIDS, is difficult to measure. Clinical data are accumulating on fluctuations in antigenaemia in infected patients throughout the incubation period (Pederson *et al.* 1987) and one hypothesis is that the peaks in antigenaemia correlate with peaks in infectiousness (Albert *et al.* 1988). As yet, data to support this hypothesis are limited (see Laurian *et al.* 1989) and a growing body of evidence points to wide variation in the likelihood that the virus is transmitted during a single sexual encounter. One approach to the study of infectiousness is to monitor the sexual partners of infected individuals to ascertain the likelihood of transmission to a susceptible partner at various time periods after seroconversion of the initially infected partner. Few such studies have been undertaken so far (table 3). In some cases the virus was transmitted after a single or few sexual contacts whereas in other cases no transmission occurred despite a high frequency of contact (Peterman *et al.* 1987).

Longitudinal studies of fluctuation in HIV antigen concentration in serum from infected patients provide some clues to what might generate such variability. Analyses of the available data on temporal fluctuation in antigen concentrations (assumed to reflect, in some manner, viral abundance and possibly infectiousness) reveal low concentrations (if at all detectable) before seroconversion (about 30–40 days), high concentrations during primary HIV infection (which has an average incubation period of around 40 days) which remain high for a few months before falling to low levels during the asymptomatic phase of the infection. As patients progress to persistent generalized lymphadenopathy (PGL) and AIDS-related complex (ARC) and finally to AIDS, antigen concentration (particularly the p24 core antigen) rises again to high levels.

This pattern suggests tentatively that there are two periods of peak infectiousness (assuming that antigen concentration reflects infectiousness), one shortly after infection lasting for a few months, and one as patients progress to ARC and AIDS, perhaps lasting for a few years (Pedersen *et al.* 1987; Anderson 1988*a*; Anderson & May 1988). It is important to remember, however, that the incubation period of AIDS is very variable between infected individuals and in some people antigen remains detectable and often at high levels of concentration throughout the course of infection. In these patients disease appears to progress rapidly (Moss 1988; Allain *et al.* 1987; Goudsmit *et al.* 1986). The hypothesis of two periods of peak infectiousness remains tentative at present.

The most striking feature of clinical data on antigen fluctuation, and of epidemiological data on the likelihood of transmission, is the high degree of variability between patients or sexual partners (Anderson & Medley 1988).

(d) *Transmission probability*

For the purpose of the models discussed in this paper we define the transmission probability, β, as the average probability that an infected individual will infect a susceptible partner over the duration of their relationship (i.e. defined per partner as opposed to per sexual contact). The form of this definition is in part determined by the character of the epidemiological data used to derive crude estimates of the likelihood of transmission, and in part by the structure of the simple mathematical models employed to study transmission (see Anderson *et al.* 1986; May & Anderson 1988). Data on the magnitude of β is limited at present and that which has

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TABLE 3. PROBABILITY OF HETEROSEXUAL TRANSMISSION, p
(n = sample size)

risk group of index case	p	n	reference
	male to female		
transfusion	0.16	50	Peterman *et al.* (1988)
transfusion	0.18	55	Peterman & Curran (1986)
mixed including bisexuals	0.07	217	Garcia *et al.* (1988)
mixed including bisexuals	0.23	97	Padian *et al.* (1987)
mixed including bisexuals	0.24	132	Padian *et al.* (1988)
mixed including bisexuals	0.24	41	Johnson *et al.* (1988)
mixed including bisexuals	0.28	104	De Vincenzi *et al.* (1988)
mixed including bisexuals	0.28	18	Roumelioutou-Karayannis *et al.* (1988)
mixed including bisexuals	0.40	68	Stasewski *et al.* (1988)
mixed including bisexuals	0.45	114	Steigbigel *et al.* (1988)
mixed including bisexuals	0.50	28	Fischl *et al.* (1988)
mixed including bisexuals	0.55	75	Sion *et al.* (1988)
African connections	0.53	62	Laga *et al.* (1988)
patients from Africa	0.61	150	Hira *et al.* (1988)
patients from Africa	0.71	14	Sewankambo *et al.* (1987)
patients from Africa	0.73	38	Taelman *et al.* (1987)
army personnel	0.33	18	Anderson & May (1988)
IV drug users	0.17	60	Johnson *et al.* (1988)
IV drug users	0.33	6	Roumelioutou-Karayannis *et al.* (1988)
IV drug users	0.35	48	Anderson & May (1988)
IV drug users	0.38	71	Milazzo *et al.* (1988)
IV drug users	0.48	88	Anderson & May (1988)
haemophiliac	0.00	36	Brettler *et al.* (1988)
haemophiliac	0.03	30	Miller *et al.* (1987)
haemophiliac	0.04	77	Jones *et al.* (1985)
haemophiliac	0.04	25	Roumelioutou-Karayannis *et al.* (1988)
haemophiliac	0.05	21	Lawrence *et al.* (1988)
haemophiliac	0.06	33	Jason *et al.* (1986)
haemophiliac	0.07	148	Allain *et al.* (1986)
haemophiliac	0.10	164	Kamradt *et al.* (1988)
haemophiliac	0.10	21	Kreiss *et al.* (1985)
haemophiliac	0.10	40	Biberfeld *et al.* (1986)
haemophiliac	0.11	35	Lawrence *et al.* (1988)
haemophiliac	0.13	124	Goedert *et al.* (1988)
haemophiliac	0.17	21	Winkelstein *et al.* (1986)
haemophiliac	0.17	24	Goedert *et al.* (1987)
	female to male		
transfusion	0.08	25	Peterman *et al.* (1988)
army personnel	0.33	6	Anderson & May (1988)
African connections	0.13	16	Laga *et al.* (1988)
patients from Africa	0.40	10	Taelman *et al.* (1987)
patients from Africa	0.73	78	Hira *et al.* (1988)
IV drug users	0.08	13	Johnson *et al.* (1988)
IV drug users	0.58	12	Steigbigel *et al.* (1987)
mixed	0.00	20	Padian *et al.* (1987)
mixed	0.04	27	De Vincenzi *et al.* (1988)
mixed	0.09	30	Stasewski *et al.* (1988)
mixed	0.50	14	Steigbigel *et al.* (1988)
mixed	0.71	17	Fischl *et al.* (1987)

been published is characterized by a high degree of variability between studies (table 3). Estimates range from less than 10% to over 60%. This variability probably reflects factors such as difference in the frequency and type of sexual contact between partnerships, and in the timing of contact during the long incubation period of the disease in the index case.

As defined above, β is an average value for the likelihood of transmission over the duration

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of the incubation period of AIDS. If there are two peak periods of infectious, one early and one late in the progression of the disease, it may be better to consider three separate values of β (i.e. β_i) for the first peak in infectiousness, the asymptomatic phase and the late peak of infectiousness, respectively. An additional variable is the average time, T_i, spent in each of these three phases of disease progression.

(*e*) *Sexual activity*

The frequency distribution of the rate at which an individual acquires new sexual partners per unit of time (e.g. one year) is an important determinant of the pattern and rate of spread of HIV in a defined community. Simple models of the transmission dynamics of HIV, based on the assumption of proportional mixing (see Hethcote & Yorke 1984; Anderson *et al.* 1986) suggest that the effective rate of partner change, c, should be defined as the mean of the distribution, m, plus the variance to mean ratio, σ^2/m (May & Anderson 1987):

$$c = m + \sigma^2/m. \tag{2}$$

This definition makes clear the importance of an understanding of the full distribution of the rate of sexual partner change in any analysis of the course of the epidemic. Various studies, based on interviews and the completion of confidential questionnaires, have attempted to define this distribution in samples drawn at various time points, from various male homosexual communities (Fay *et al.* 1989). Sampling procedures have ranged from quota to random methodologies (see Anderson (1988*b*) and Anderson & Johnson (1989) for recent reviews of published studies). Survey design and sampling methods for the collection of data on rates of sexual partner change present many problems (Cohen 1987). The studies in the U.K. that have been published so far reveal great variability both in type of sexual activity and in partner change rate, with dependencies on variables such as age, social status and the geographical location from which the sample was drawn (McKusick *et al.* 1985; Carne *et al.* 1985; British Market Research Board (BMRB) 1987).

An interesting feature of the data is the existence of a general relation between the mean (m) and variance (σ^2) of the recorded distributions of sexual partner change of power law from:

$$\sigma^2 = am^b. \tag{3}$$

The coefficients a and b can be estimated by regression techniques from a logarithmic plot of σ^2 against m for all published values of the two statistics. A recent analysis of this relation, based on studies that made use of a wide range of sampling methods, sampled different populations and recorded numbers of different sexual partners over differing time intervals (ranging from the previous month to lifetime), revealed a linear relation on a logarithmic scale with coefficient values $a = 0.555$, $b = 3.231$ (Anderson & May 1988). Why there should be such a tight relation between the two statistics, irrespective of sampling method and sample population, is unclear at present (figure 1). However its existence enables us to express (2) in the form

$$c = m + am^{b-1} \tag{4}$$

A summary of the mean rates of sexual partner change recorded in studies of male homosexual communities in the U.K. is presented in table 4.

The focus in models of HIV transmission on the rate of sexual partner change glosses over

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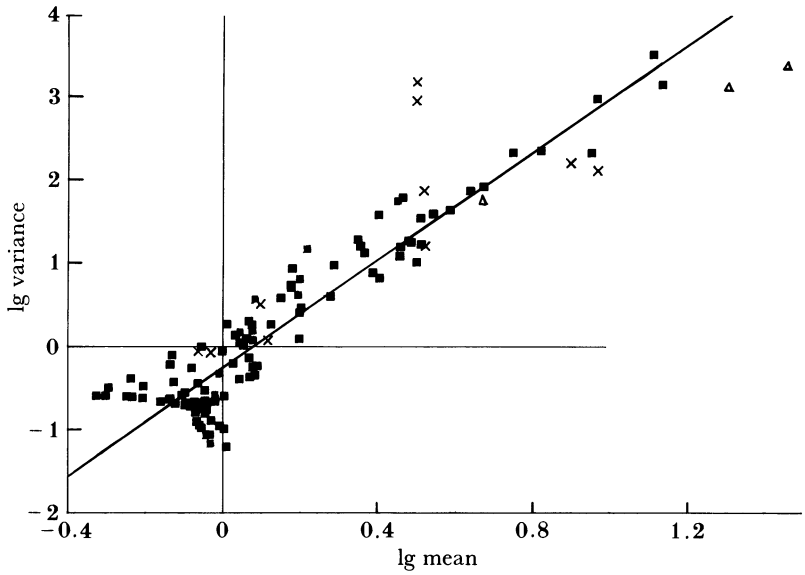

FIGURE 1. A scatter plot of the logarithm (base 10) of the variance in a reported number of different sexual partners against the logarithm of the mean number of sexual partners, from a wide range of published and unpublished studies (see Anderson & May 1988). The solid line denotes the best fit power model of the form defined by equation (4) in the main text. The squares record data from studies of heterosexuals in developed countries, the crosses are data from studies of homosexual males and the triangles are from studies of heterosexuals in developing countries.

TABLE 4. NUMBER OF SEXUAL PARTNERS PER TIME UNIT

(n = sample size)

population	n	time unit	mean number of partners mean	median	reference
		heterosexual and general			
U.K. (m+f), BMRB	—	per year	0.92	—	Anderson (1988)
U.K. (m+f), Harris poll	823	per year	1.50	—	Anderson (1988)
U.K. Students (m+f)	41	per year	1.50		Anderson (1988)
U.S.A. (m+f)	346	past year	6.3		Fischl *et al.* (1988)
		past 5 years	14.9	—	Fischl *et al.* (1988)
U.S.A. (m+f)	851	past 3 months	1.49		Baldwin & Baldwin (1988)
		past year	2.11		Baldwin & Baldwin (1988)
U.S.A. (m)	—	1985	2.84	—	Guydish & Coates (1988)
		1987	1.7	—	Guydish & Coates (1988)
African (m+f)	84	lifetime	7.5	—	Cornet *et al.* (1988)
Zaire, AIDS (m)	20	past year	7	—	Piot *et al.* (1984)
Zaire, AIDS (f)	18	past year	3	—	Piot *et al.* (1984)
Zaire, HIV+ couples	257	per year	5.5	—	Nzila *et al.* (1988)
(f)	257	per year	1.0	—	Nzila *et al.* (1988)
rural Uganda (f)	126	past 5 years	1.2	—	Hudson *et al.* (1988)
10–14	24	past 5 years	0.3	—	Hudson *et al.* (1988)
15–19	19	—	0.8	—	Hudson *et al.* (1988)
20–34	51	—	1.5	—	Hudson *et al.* (1988)
35–49	21	—	1.0	—	Hudson *et al.* (1988)
50+	11	—	0.5	—	Hudson *et al.* (1988)
rural Uganda (m)					
10–14	23	—	1.0	—	Hudson *et al.* (1988)
15–19	19	—	3.8	—	Hudson *et al.* (1988)
20–34	42	—	2.8	—	Hudson *et al.* (1988)
35–49	20	—	3.5	—	Hudson *et al.* (1988)
50+	19	—	1.9	—	Hudson *et al.* (1988)

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TABLE 4. (*Cont.*)

population	*n*	time unit	mean number of partners: mean	mean number of partners: median	reference
East African pastoralist (m)	132	per year	11.8	—	Konings *et al.* (1989)
Rwanda (m)	27	per year	3.0		Van De Perre *et al.* (1985)
		female partners of bisexual men			
U.S.A.	169	past 2 years	2.9	—	Winkelstein *et al.* (1986)
U.S.A.	123	past 6 months, 1984	2.81	—	Winkelstein *et al.* (1987)
		past 6 months, 1986	1.81	—	Winkelstein *et al.* (1987)
Brazil HIV+	45	past 2 years	3.0	—	Costa *et al.* (1988)
Brazil HIV−	133	past 2 years	3.0	—	Costa *et al.* (1988)
		male homosexual partners			
U.K.	100	per month 1984/5	4.7	3	Carne *et al.* (1987)
		per month 1986	—	1	Carne *et al.* (1987)
U.K. BMRB	—	per year	8.7	—	Anderson (1988)
	156	per 3 months	3.0	—	Anderson (1988)
		per year	10.5	—	Anderson (1988)
	298	per 3 months	3.7	—	Anderson (1988)
		per year	8.7	—	Anderson (1988)
	284	per 3 months	3.1	—	Anderson (1988)
		per year	7.1	—	Anderson (1988)
	251	per 3 months	2.0	—	Anderson (1988)
		per year	4.8	—	Anderson (1988)
Northern Ireland	30	per year 1984	16.6	—	Maw *et al.* (1987)
		per year 1985/6	20.8	—	Maw *et al.* (1987)
Belgium	526	1984	14.4	—	Goilav *et al.* (1988)
		1986	12.2	—	Goilav *et al.* (1988)
Netherlands	940	past 6 months	19.1	—	Countinho *et al.* (1987)
Italy	75	per month	—	10	Tirelli *et al.* (1988)
U.S.A. SF	655	1982	3.7	—	McKusick *et al.* (1985)
U.S.A. SF	454	1983	3.2	—	McKusick *et al.* (1985)
U.S.A. SF (steady)	126	1978	1.6	—	CDC (1987)
		1984	2.5	—	CDC (1987)
		1985	1.5	—	CDC (1987)
U.S.A. SF (non-steady)	126	past 4 months 1978	29.3	16	CDC (1987)
		past 4 months 1984	14.5	3	CDC (1987)
		past 4 months 1985	5.5	1	CDC (1987)
U.S.A.	78	per year	—	27	Jaffe *et al.* (1983)
U.S.A.	42	per year	—	25	Jaffe *et al.* (1983)
U.S.A. Kaposi's sarcoma and PCP patients	50	per year	61	—	Jaffe *et al.* (1983)
U.S.A. aware of HIV status	670	past 6 months	3.5	—	Fox *et al.* (1987)
U.S.A. unaware of HIV status	331	past 6 months	3.9	—	Fox *et al.* (1987)
U.S.A. NY	745	per year, pre 1985	—	36	Martin (1987)
		1985	8	—	Martin (1987)
U.S.A. Chicago	637	per month	1.74	—	Joseph *et al.* (1987)
U.S.A. Madison	488	past month 1982	6.2	—	Golubjatanikov *et al.* (1983)
		past month 1983	3.2	—	Golubjatanikov *et al.* (1983)
U.S.A. MACS HIV+	95	past 6 months	10	—	Kingsley *et al.* (1987)
		past 2 years	—	36	Kingsley *et al.* (1987)
		lifetime	—	200	Kingsley *et al.* (1987)
U.S.A. MACS HIV−	2412	past 6 months	—	5	Kingsley *et al.* (1987)
		past 2 years	—	20	Kingsley *et al.* (1987)
		lifetime	—	100	Kingsley *et al.* (1987)

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TABLE 4. (*Cont.*)

population	*n*	time unit	mean number of partners mean	median	reference
Brazil HIV+	45	past 2 years	—	30	Costa *et al.* (1988)
HIV−	133	past 2 years	—	10	Costa *et al.* (1988)
		customers of prostitutes			
Rwanda (m)	25	per year	—	31	Van De Perre *et al.* (1985)
		prostitutes (f)			
U.S.A. NY	78	past year	256	200	Seidlin *et al.* (1988)
Netherlands	117	per month	84	—	Van den Hoek *et al.* (1988)
Brazil	113	per year	800	—	Castello-Branco *et al.* (1988)
Kenya	429	per day 1985	3.7	—	Plummer *et al.* (1988)
HIV+	261	per day 1985	3.8	—	Plummer *et al.* (1988)
HIV−	168	per day 1985	3.6	—	Plummer *et al.* (1988)
using oral contraceptive	120	per day 1985	4.0	—	Plummer *et al.* (1988)
not using oral contraceptive	309	per day 1985	3.5	—	Plummer *et al.* (1988)
highly educated	78	per day 1985/6	3.8	—	Plummer *et al.* (1988)
medium educated	82	per day 1985/6	3.9	—	Plummer *et al.* (1988)
low educated	205	per day 1985/6	5.7	—	Plummer *et al.* (1988)
Kenya	124	per day 1985/7	3.8	—	Plummer *et al.* (1988)
HIV+	83	per day 1985/7	4.1	—	Plummer *et al.* (1988)
HIV−	41	per day 1985/7	3.3	—	Plummer *et al.* (1988)
Kenya low class	64	per year	963	—	Kreiss *et al.* (1986)
high class	26	per year	124	—	Kreiss *et al.* (1986)
Kenya HIV+	5	per month 1981	180	—	Piot *et al.* (1987)
Kenya HIV+	70	per month 1984/5	123	—	Piot *et al.* (1988)
HIV−	111	per month 1981	54	—	Piot *et al.* (1988)
HIV−	57	per month 1984/5	117	—	Piot *et al.* (1988)
Zaire	377	past week	3.7	—	Mann *et al.* (1988)
		past month	15.9	—	Mann *et al.* (1988)
		past year	158.0	—	Mann *et al.* (1988)
		lifetime	703.0	—	Mann *et al.* (1988)
HIV+	—	lifetime	—	600	Mann *et al.* (1988)
HIV−	—	lifetime	338	—	Mann *et al.* (1988)
Nigeria	823	per day	3.3	—	Mohammed *et al.* (1988)
Nigeria	35	per month	—	3.0	Ayoola *et al.* (1988)
Nigeria	767	per day	3.3	—	Chikwem *et al.* (1988)
		per year	1046	—	Chikwem *et al.* (1988)
Rwanda	33	per month	—	44	Van De Perre *et al.* (1985)
Somalia	55	per week	14	—	Jama *et al.* (1987)
		prostitutes (m)			
Italy, IV drug	27	per month	—	50	Tirelli *et al.* (1988)
Italy, IV drug	76	per month	—	100	Tirelli *et al.* (1988)
Brazil	24	per year	10	—	Castello-Branco *et al.* (1988)
Brazil, transvestites	16	per year	480	—	Castello-Branco *et al.* (1988)
		STD patients			
Canada (f)	707	past year	—	12	Elmslie *et al.* (1988)
Kenya (f)	40	past year	17	—	Kreiss *et al.* (1986)
Kenya (f), HIV+	8	per month	1.2	—	Piot *et al.* (1987)
Kenya (f), HIV−	113	per month	1.2	—	Piot *et al.* (1987)
Kenya (m), HIV+	36	per month	1.5	—	Piot *et al.* (1987)
Kenya (m), HIV−	232	per month	1.1	—	Piot *et al.* (1987)
Kenya (m)	115	past 6 weeks	1.7	—	Greenblatt *et al.* (1988)
Kenya (m), HIV+	19	past 6 weeks	1.4	—	Greenblatt *et al.* (1988)
Kenya (m), HIV−	96	past 6 weeks	1.8	—	Greenblatt *et al.* (1988)

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much detail concerning variability in type of sexual activity between partnerships (e.g. oral or anal sex, etc.). Extensive studies of a large cohort of 4955 homosexual and bisexual men from the Baltimore, Chicago, Los Angeles and Pittsburgh areas of the U.S.A. (the Multicentre AIDS Cohort Study (MACS)) that started in 1983, revealed that the number of different sexual partners is, overall, the most important factor in determining the likelihood of HIV infection. However, among those with high rates of partner change, other factors such as the frequency of receptive and insertive anal intercourse are of great significance (Ginzburgh *et al.* 1988). A total of 8% of the 2915 individuals who were seronegative at the start of the study acquired infection over a two-year period. The rate of seroconversion among men who reported practising receptive but not insertive anal intercourse was 3.6 times higher than among men practising insertive intercourse alone, despite reports of 38% more different partners among those who practised insertive intercourse only.

(*f*) *Changes in sexual behaviour*

Education and a knowledge of the devastating impact of HIV infection, via friendship and association with infected individuals, has resulted in significant changes in sexual behaviour among male homosexuals in countries such as the U.S.A. and the U.K. Evidence for such changes comes indirectly from a decline in the incidences of other sexually transmitted infection such as gonorrhoea (Carne *et al.* 1986; Gellan & Ison 1986), and directly from longitudinal surveys of rates of sexual partner change, type of sexual activity and the frequency of practice of 'safe-sex' methods (Ginzburgh *et al.* 1988). Personal acquaintance with someone who has AIDS appears to be a powerful motivator for a change in sexual behaviour.

Unfortunately, no single study has charted changes in rates of sexual partner change, year by year, over the course of the epidemic. One study at four different sampling time points between February 1986 and January 1987 in the U.K. revealed a 50% reduction in claimed rates of partner change over the one-year time interval (see figure 2 and BMRB (1987)). More generally, however, the degree to which habits have changed in the male homosexual community in the U.K. from 1978 to the present, remains a matter of speculation (Evans *et al.* 1989). Reported declines in the incidences of other sexually transmitted diseases among male homosexuals suggest changes in behaviour were beginning to have an impact on disease

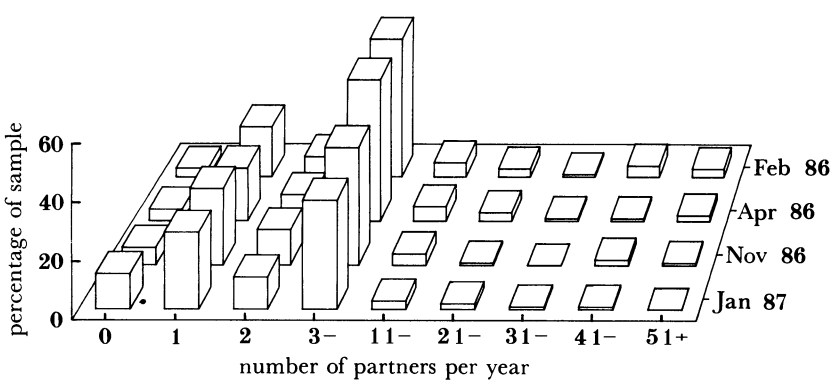

FIGURE 2. Patterns of change in the frequency distribution of reported numbers of different sexual partners per year in samples of male homosexuals in England drawn in February (10.5), April (8.7) and November 1986 (7.1) and January 1987 (4.8) (data from BMRB 1987). The mean rate of partner change per annum declined from 10.5 in February 1986 to 4.8 in January 1987.

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transmission in the early 1980s (Weller *et al.* 1984; Johnson & Gill 1989; Loveday *et al.* 1989). It must be stressed, however, that accurate prediction of future trends in the incidence of AIDS in male homosexuals requires a quantitative understanding of temporal changes in sexual behaviour.

3. Mathematical models

The general model of HIV transmission dynamics in a one-sex population with heterogeneity in sexual activity is essentially that first proposed by Anderson *et al.* (1986),

$$\frac{\partial x(s,t)}{\partial t} = \Lambda(s,t) - [\lambda(s,t)\, s + \mu]\, x(s,t), \tag{5}$$

$$\frac{\partial y(u,s,t)}{\partial t} + \frac{\partial y(u,s,t)}{\partial u} = -[v(u) + \mu]\, y(u,s,t), \tag{6}$$

with boundary and initial conditions:

$$x(s,t_0) = \Lambda(s,t_0)/\mu, s > 0, \tag{7}$$

$$y(0,s,t) = \lambda(s,t)\, s x(s,t), t > t_0, \tag{8}$$

$$y(u,s,t_0) = \eta(u,s), \tag{9}$$

where

s = sexual activity (new partners per unit time),

t = time,

t_0 = time at which infected person(s) entered the population,

$\int_a^b x(s,t)\, \mathrm{d}s$ = number of uninfected people with activity between a and b, at time t,

$\int_c^d \int_a^b g(u,s,t)\, \mathrm{d}s\, \mathrm{d}u$ = number of infected people with activity between a and b, who become infected between times $t-c$ and $t-d$, at time t,

$\Lambda(s,t)$ = rate of recruitment of newly sexually active people to the uninfected population, with activity s, at time t. The activity of an individual does not change with time,

μ = reciprocal of average duration of a sexual 'lifetime',

u = time since becoming infected,

$v(u)$ = fractional loss rate, per unit time, of individuals who have been infected for a time u. Losses occur as individuals develop AIDS. The hazard function $v(u)$ is

related to the probability density function for the incubation period of AIDS, $f(u)$, by

$$f(u) = v(u) \exp\left\{-\int_0^u v(x)\,\mathrm{d}x\right\}, \tag{10}$$

$\int_c^d \int_a^b \eta(u,s)\,\mathrm{d}s\,\mathrm{d}u =$ the initial number of infected people (i.e. at $t = t_0$), with activity between a and b, who have been infected for a time between c and d,

$\lambda(s,t)\,s =$ fraction per unit time of uninfected people with activity s who become infected at time t (the so-called force of infection). For variable infectiousness, we write

$$\lambda(s,t) = \int_0^\infty \rho(s,r,t) \frac{\int_0^\infty \beta(u)\,y(u,r,t)\,\mathrm{d}u}{x(r,t) + \int_0^\infty y(u,r,t)\,\mathrm{d}u}\,\mathrm{d}r, \tag{11}$$

where

$\beta(u) =$ infectiousness of someone who has been infected for a time u, i.e. the probability per unit, time of being infected through sexual contact with such a person. Note that if $\beta(u)$ is a constant β, then $\lambda(s,t)$ reduces to the integral of $\rho(s,r,t)$ times the fraction of people with activity r who are infectious (at time t), times β,

$\rho(s,r,t) =$ the fraction of the partners which someone of activity s taken among people with activity r, i.e. the 'contact function'.

Note that in this formulation, no account is taken of the number of times sexual intercourse occurs between partners.

We now outline some specific details of the model, and certain approximations that facilitate numerical analysis.

(a) *Recruitment*

We approximate the recruitment function $\Lambda(s,t)$ by the function $\Lambda W[s, a(t), b(t)]$, where Λ is the total number of sexually active individuals entering the susceptible population per unit time, and W is a Weibull probability density function describing the distribution of sexual activity of these recruits:

$$W(s, a(t), b(t)) \equiv b(t)\,a(t)^{b(t)} s^{b(t)-1} \exp\{-(a(t)\,s)^{b(t)}\}. \tag{12}$$

The mean and variance of the sexual activity of recruits is therefore given by:

$$\mathrm{m}(t) = \frac{1}{b(t)} \Gamma\left(1 + \frac{1}{b(t)}\right) \tag{13}$$

and

$$\mathrm{var}\,(t) = \mathrm{m}(t) \left[\frac{1}{\Gamma(2 + b(t) - 1)}\right] \tag{14}$$

respectively, where $\Gamma(x)$ is the gamma function

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du. \tag{15}$$

We simplify matters somewhat by assuming that the empirical relation between mean and variance of sexual activity described earlier (see Anderson & May 1988) holds true, so that (see equation (3)):

$$\text{var}(t) = a\text{m}(t)^b. \tag{16}$$

We can then treat W as a function with one parameter only, by taking a given mean activity $\text{m}(t)$, calculating var (t) from (16), and using (13) and (14) to obtain the approximate values of $a(t)$ and $b(t)$. We therefore use the shorthand notation $W[s, m(t)]$.

(*b*) *Incubation period and infectiousness*

Major reductions in the computations associated with solving the model can be made by careful choice of $f(u)$ and $\beta(u)$ (see Blythe & Anderson 1988*a*, *b*). We express the incubation period probability density function as the sum of three exponential terms:

$$f(u) = \sum_{j=1}^{3} a_j \exp(-\sigma_j u), \quad j = 1, 2, 3, \tag{17}$$

where

$$a_j = \prod_{i=1}^{3} \sigma_j \Big/ \prod_{\substack{i=1 \\ i \neq j}}^{N} (\sigma_i - \sigma_j), \quad j = 1, 2, 3, \tag{18}$$

and the σ_j are the reciprocals of the average duration of each exponentially distributed sub-stage. By assigning a fixed value β_j to each sub-stage, we may replace (10) by the set of three simple equations:

$$\frac{\partial y_1(s,t)}{\partial t} = \lambda(s,t)\, sx(s,t) - (\sigma_1 + \mu)\, y_1(s,t), \tag{19}$$

$$\frac{\partial y_i(t)}{dt} = \sigma_{j-1} y_{j-1}(s,t) - (\sigma_j + \mu)\, y_j(s,t), \quad j = 2, 3. \tag{20}$$

With initial conditions

$$y_1(s, t_0) = \int_0^\infty \eta(u, s)\, du, \tag{21}$$

$$y_j(s, t_0) = 0, \quad j = 2, 3, \tag{22}$$

making the simplifying assumption that the invading infected individuals are all in the initial infectious sub-stage.

The incidence of AIDS (rate at which new cases are diagnosed per unit time) is then

$$V(t) = \sigma_3 \int_0^\infty y_3(s,t)\, ds, \tag{23}$$

while we may write

$$\lambda(s,t) = \int_0^\infty \rho(s,r,t) \frac{\sum_{j=1}^{3} \beta_j y_j(r,t)}{N(r,t)} dr, \tag{24}$$

where

$$N(r,t) = x(r,t) + \sum_{j=1}^{3} y_j(r,t); \tag{25}$$

$\int_a^b N(r,t)\, dr$ is the total number of people with activity between a and b at time t.

Following Blythe & Anderson (1988*b*), we shall assume that the infectiousness of an individual is relatively high immediately after infection occurs ($\beta_1 > 0$), is effectively zero for a time ($\beta_2 = 0$), and rises again as a diagnosis of AIDS is approached ($\beta_3 > 0$).

(*c*) *Contact function*

The function $\rho(s, r, t)$ describes the mixing between individuals of different activity, and must satisfy the three constraints:

$$1 \geqslant \rho(s, r, t) \geqslant 0, s, r, t, \tag{26}$$

$$\int_0^\infty \rho(s, r, t)\, \mathrm{d}r = 1, \forall\, s, t, \tag{27}$$

$$sN(s, t)\, \rho(s, r, t) = rN(r, t)\, \rho(r, s, t), \forall\, s, r, t. \tag{28}$$

Equations (26) and (27) reflect the fact that ρ is effectively a probability density function, whereas (24) expresses the requirement that the total number of partnerships between s-people (i.e. those with activity s) and r-people must equal the total number of partnerships between r-people and s-people. It is expected that ρ must lie somewhere between two limiting cases. At one extreme, individuals (of any activity) take their partners at random from the population. In this case the activity level with the most number of partnerships (i.e. $rN(r, t)$) is most strongly represented, and ρ becomes a function of r and t alone, namely:

$$\rho(s, r, t) = \frac{rN(r, t)}{\int_0^\infty zN(z, t)\, \mathrm{d}z}. \tag{29}$$

This case is known as 'proportionate mixing'. An example of $\rho(s, r)$ (for constant population size $\int_0^\infty N(s)\, \mathrm{d}s$) is shown in figure 3*a*.

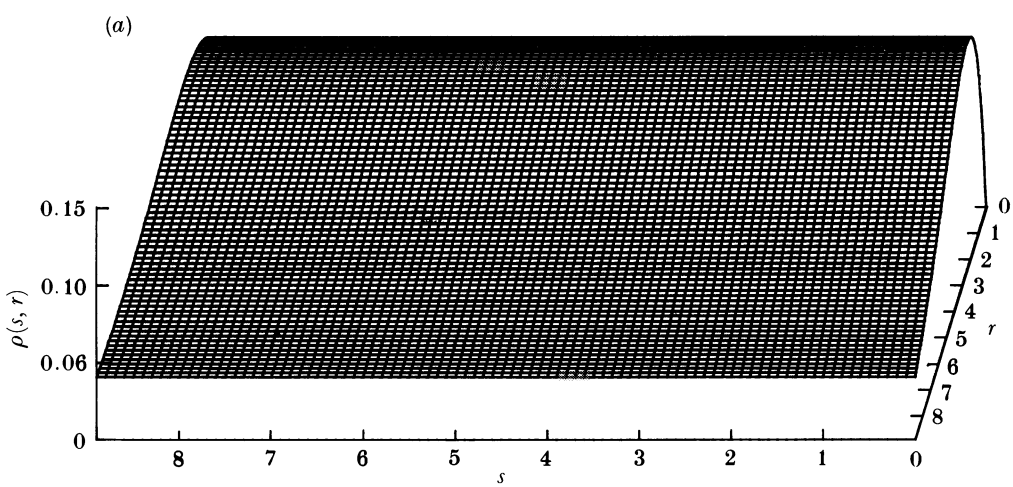

FIGURE 3. For legend see opposite.

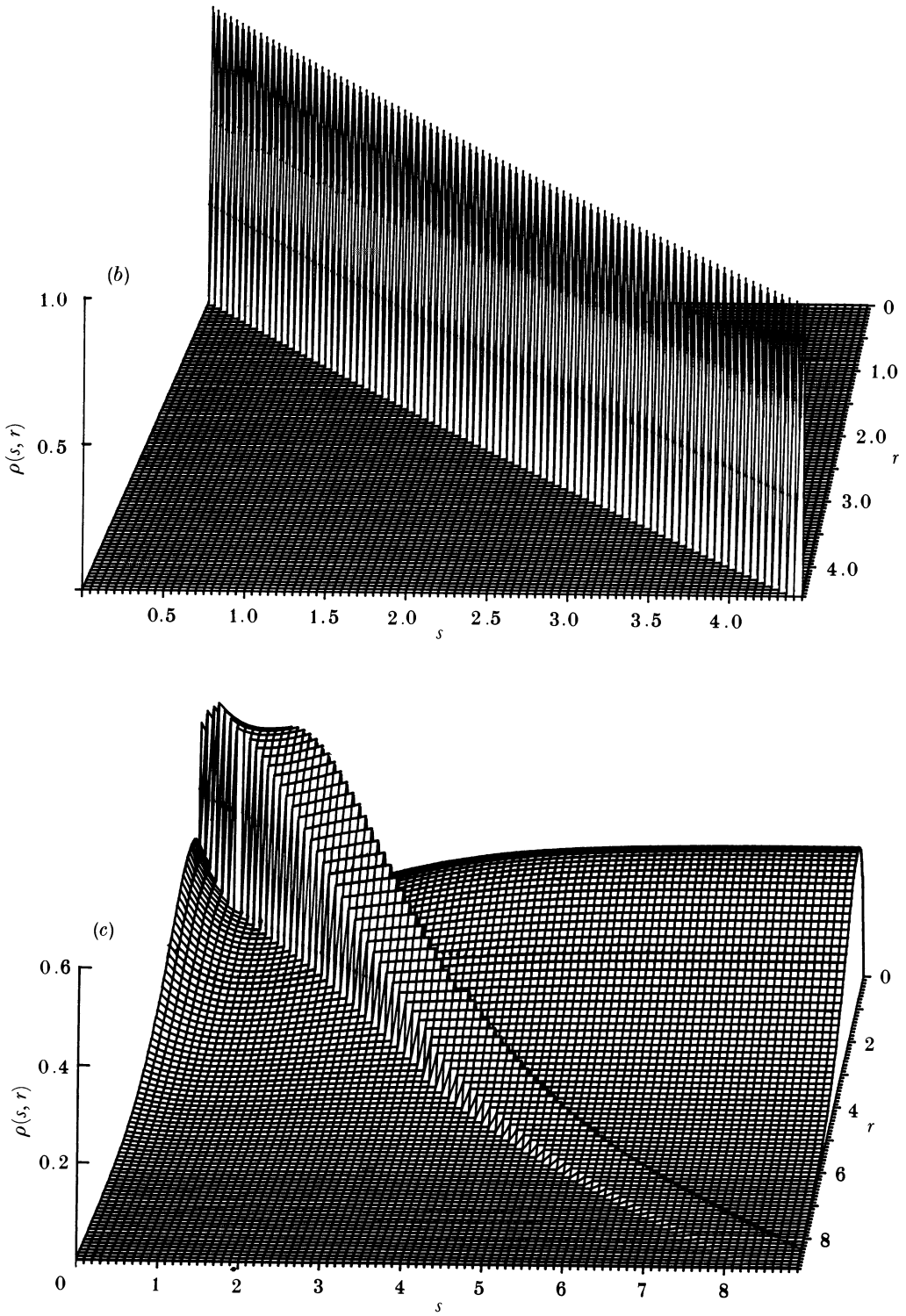

FIGURE 3. Contact functions. (*a*)The function $\rho(s, r, t)$ describes the mixing between individuals in different sexual activity classes. The case illustrated is for proportionate mixing where s and r denote the activity class of each individual in any given partnership (see text). (*b*) The extreme case where individuals only chose partners from their own activity level such that $\rho(s, r, t)$ becomes a delta function with $s = r$. (*c*) A mixture of proportionate mixing and preferred mixing within a given activity class (see Jacquez *et al.* 1988).

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At the other extreme, individuals only select partners of their own activity level, so that $\rho(s, r, t)$ becomes a delta-function at $s = r$ (schematically represented in figure 3*b*). In this case (11) becomes:

$$\lambda(s,t) = \frac{\int_0^\infty \beta(u)\, y(u,s,t)\, \mathrm{d}u}{x(s,t) + \int_0^\infty y(u,s,t)\, \mathrm{d}u}, \tag{30}$$

so that there is no mixing between individuals from different activity levels. We do not believe that this case has any practical relevance. It simply serves to define the extreme case of preferential mixing.

The proportionate mixing solution has been used in a variety of situations (e.g. Barbour 1978; Nold 1980; Anderson & May 1984; Hethcote & York 1984; Dietz & Schenzle 1985; Anderson & Grenfell 1986; Hethcote & van Ark 1987; Blythe & Anderson 1988*c*; Castillo-Chavez *et al.* 1988); while recently Sattenspiel (1987*a*, *b*); Sattenspiel & Simon (1988); Jacquez *et al.* (1988) have explored alternative forms of mixing. Stanley (1989) and Blythe & Castillo-Chavez (1989) have also found new forms of ρ which satisfy the constraints given in equations (26)–(28) (figure 3*c*).

In this study we have not attempted to investigate the effects of preferential mixing, but instead chose to use the proportionate mixing model (equation (29)). Under these circumstances, we write:

$$\lambda(s,t) = \lambda(t) \equiv \int_0^\infty r \sum_{i=1}^{3} \beta_i y_i(r,t)\, \mathrm{d}r \Big/ \int_0^\infty r N(r,t)\, \mathrm{d}r. \tag{31}$$

(*d*) *Aggregation approximation*

Even in the simplified form resulting from the approximations outlined above, the transmission model still consists of three partial differential equations containing two integrals. To reduce the burden of computation further (which is advantageous if, for example, a sensitivity analysis is to be done), we introduce an extended version of Blythe & Anderson's (1988*c*) discretization scheme for the variable infectiousness model. This scheme is loosely related to the approximations of aggregation theory (see Iwasa 1987), and replaces the integro-partial differential equation system with a set of ordinary differential equations, whose behaviour should closely approximate that of the original system in defined ways (e.g. same steady states, stability properties, behaviour at $t \simeq t_0$, etc.).

Following Blythe and Anderson (1988*c*) we subdivide the continuum of sexual activity s into N discrete 'activity classes', with boundaries $\{s_i\}$, an ordered set with $s_{N+1} = 0$ and $s_{N+1} = \infty$, and characteristic levels of sexual activity $\{c_i\}$. The model system may then be written as

$$\mathrm{d}X_i(t)/\mathrm{d}t = \Lambda_i(t) - [\lambda(t)\, c_i + \mu]\, X_i(t), \tag{32}$$

$$\mathrm{d}Y_{i,1}(t)/\mathrm{d}t = \lambda(t)\, c_i\, X_i(t) - (\sigma_1 + \mu)\, Y_{i,1}(t), \tag{33}$$

$$\mathrm{d}Y_{i,j}(t)/\mathrm{d}t = \sigma_{j-1}\, Y_{i,j-1}(t)\, (\sigma_j + \mu)\, Y_{i,j}(t), \tag{34}$$

for $i = 1, 2, \ldots, N$ and $j = 2, 3$, with initial conditions

$$X_i(t_0) = \Lambda_i(t_0)/\mu, \tag{35}$$

$$Y_{i,1}(t_0) = \int_{s_i}^{s_{i+1}} \int_0^\infty \eta(u,s)\, \mathrm{d}u\, \mathrm{d}s, \quad Y_{i,j}(t_0) = 0, \tag{36}$$

for $i = 1, 2, \ldots, N, j = 2, 3$. We have made the simplifying assumption that the initial infected individuals are all in the first infectious stage ($j = 1$). Equation (36) indicates that $Y_{j,1}(0) =$ constant. In addition we have:

$$\Lambda_i(t) = \int_{s_i}^{s_{i+1}} \Lambda(s, t)\, \mathrm{d}s, \tag{37}$$

which is easily derived from (12)–(16), and

$$\lambda(t) = \frac{\sum_{i-1}^{N} c_i \sum_{j=1}^{3} \beta_j Y_{i,j}(t)}{\sum_{i=1}^{N} c_i \left[X_i(t) + \sum_{j=1}^{3} Y_{i,j}(t) \right]}. \tag{38}$$

Following Blythe & Anderson (1988*c*), the $\{c_j\}$ are chosen according to a scheme which is outlined below, with further details given in appendix A. $N-1$ of the $\{c_i\}$ are set in such a way as to give the same value for the steady-state susceptible class populations as for the equivalent section of the continuous s model, i.e.

$$X_i^* = \int_{s_i}^{s_{i+1}} x^*(s)\, \mathrm{d}s, \quad i \neq i_B. \tag{39}$$

The remaining i_B^{th} class has an activity level c_{iB} set by requiring that the conditions for endemic persistence of infection is the same in the two models.

The addition of variable infectiousness adds little to the original scheme of Blythe & Anderson (1988*c*), but the change in sexual behaviour expressed in $\Lambda(s, t)$ introduces two complications.

(i) To reduce the complexity of the problem, we restrict changes in $\Lambda(s, t)$ to stepwise variation. Specifically, we use τ time intervals, each with a distinct value of mean sexual activity $\mathrm{m}(t) = \mathrm{m}_k$ ($k = 0, 1, \ldots, \tau$), and with boundaries $\{\tau_k = t\}$. Then for simplicity

$$\Lambda_{i,k}(t) = \Lambda \int_{s_i}^{s_{i+1}} W(s, m_k)\, \mathrm{d}s = \Lambda \gamma_{i,k}, \ \tau_k \leqslant t < \tau_{k+1}. \tag{40}$$

(ii) We must take account of the non-trivial steady-state infected population when $m(t)$ becomes low enough such that the endemicity condition ($R_0(m) > 1$, see Blythe & Anderson 1988*c*) is violated. When this occurs, the scheme outlined above is inappropriate, as the infected steady-states are zero in both models (and $x^*(s) = N^*(s)$), such that the $\{c_i\}$ cannot be assigned values. In this case we use the simpler but less accurate approximation (Blythe & Anderson 1988*c*):

$$c_{i,k} = \int_{s_i}^{s_{i+1}} s\gamma(s, m_k)\, \mathrm{d}s, \quad \tau_k \leqslant t < \tau_{k+1}. \tag{41}$$

(*e*) *Changes in the sexual behaviour of susceptible recruits to the sexually active population*

In the following sections (*e*), (*f*) and (*g*) we outline the methods adopted to simulate changes in sexual behaviour induced by education and a knowledge of the disease AIDS. Three types of behavioural changes are considered, namely changes in rates of sexual partner change among susceptible recruits to the sexually active adult population (*e*), changes in the magnitude of the average transmission probability β induced via the adoption of 'safer sex practices' (i.e. the use of condoms and a reduction in high-risk behaviours such as anal intercourse) (*f*) and changes in rates of sexual partner change among recruits and the sexually active population (*g*).

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Changes in the distribution of rates of acquiring new sexually active population during the course of the epidemic are induced by defining a separate distribution for new susceptibles, with a reduced mean (from that prevailing in the existing sexually active population) and a variance as defined by the power law relationship given in equation (3). As detailed in section (*d*), the continuum of the sexually active is divided into N discrete activity classes, which are chosen to match those defined for the existing sexually active adults. At any given point in time the numbers of new recruits in each activity class are added to the identical activity class in the susceptible segment of the existing sexually active population. With a reduced mean level of activity, the system will eventually settle to a new equilibrium of endemic infection (with a reduced incidence of AIDS and a reduced prevalence of HIV infection) over many decades as the new recruits with reduced sexual activity gradually replace the existing population of higher activity, as they die from AIDS or natural causes.

(*f*) *Reduction in transmission risk*

It is relatively straightforward to incorporate an approximation to the effects of a decreasing prevalence of high-risk sexual practices, such as receptive anal intercourse. We simply let the $\{\beta_i\}$ themselves become (decreasing) functions of time $\{\beta_i(t)\}$. In order for the aggregation scheme of the previous section to be workable, we restrict the time-variation in the $\{\beta_i(t)\}$ to stepwise decreases coincident with the changes in the mean number of sexual partners, m_k. Hence we have:

$$\beta_j(t) = \beta_{j,k}, \quad \tau_k \leqslant t < , t_{k+1} \quad j = 1, 2, 3, \; k = 0, 1, \ldots, \tau, \tag{42}$$

and the final approximate form of the model is

$$\mathrm{d}X_i(t)/\mathrm{d}t = \Lambda\gamma_{i,k} - [\lambda_k(t)\, c_{i,k} + \mu]\, X_i(t), \tag{43}$$

$$\mathrm{d}Y_{i,1}(t)/\mathrm{d}t = \lambda_k(t)\, c_{i,k}\, X_i(t) - (\sigma_1 + \mu)\, Y_{i,1}(t), \tag{44}$$

$$\mathrm{d}Y_{i,1}(t)/\mathrm{d}t = \sigma_{j-1}\, Y_{i,j-1}(t) - (\sigma_j + \mu)\, Y_{i,j}(t) \tag{45}$$

($j = 2, 3$), with initial values given by (35) to (36), and

$$\lambda_k(t) = \frac{\sum_{i=1}^{N} c_{i,k} \sum_{j=1}^{3} \beta_{j,k}\, Y_{i,j}(t)}{\sum_{i=1}^{N} c_{i,k} \left\{ X_i(t) + \sum_{j=1}^{3} Y_{i,j}(t) \right\}}, \tag{46}$$

with the steps in $\{\gamma_{i,k}\}$, $\{c_{i,k}\}$ and $\{\beta_{i,k}\}$ occurring at times $\{\tau_k\}$, and the $\{c_{i,k}\}$ calculated according to the aggregation scheme of (43).

(*g*) *Changes in the rate of acquiring new sexual partners within the existing sexually active population*

Changes in the distribution of sexual activity within the population may be mirrored in a variety of ways. For example, individuals in different sexual activity classes within the susceptible (X) and infected (the Y_js) populations may be rearranged such that the activity within the total population conforms to a new distribution, $g'(i)$, dictated by a desired reduction in the mean activity and its associated variance (see (3)). So if $g_0(i)$ and $g_j(i)$ are the

new distributions of activity for the susceptible and infected populations respectively, a set of solutions $\{g_j(i), j = 0, 1, 2, 3\}$ must be identified that satisfies the constraints:

$$g_0(i)\, X + \sum_{j=1}^{3} g_j(i)\, Y_j = g'(i)\, N, \tag{47}$$

$$X + \sum_{j=1}^{3} Y_j = N. \tag{48}$$

A number of solutions will satisfy these constraints and each can be translated into a particular programme of behavioural changes. For example, to satisfy a new mean and variance, changes may occur only in the susceptible population, or only among the infecteds. We consider four schemes in the results section of the paper. These are as follows.

(i) *Scheme* 1

If the probability of an individual moving to a particular sexual activity class (under a programme of change in behaviour) is independent both of the individual's infection status and of his current level of sexual activity, then the subpopulations will adopt identical distributions: $g_j(i) = g'(i), j = 0, 1, 2, 3$. As the initial distribution (before changes in behaviour) of activity among infecteds will contain (by definition) a larger than average number of individuals with high sexual activity, these 'random' changes between activity classes will result in a greater degree of change in behaviour among infecteds when compared with susceptibles.

If the probability of an individual moving to a particular sexual activity class is independent of infection status, but not of his current sexual activity, then it is necessary to make some assumptions concerning the dependence of the probability on sexual activity. We consider two possible sets of assumptions labelled schemes 2 and 3.

(ii) *Scheme* 2

We classify the sexual activity classes into those that receive individuals from other activity classes and those that lose individuals to other activity classes. We assume that individuals from the latter move into the former independent of infection status. Thus, if $G(i)$ is the gain and $L(i)$ the loss, from class i, if $G(i) > 0$, $L(i) = 0$ and if $G(i) = 0$, $L(i) > 0$, and $GX(i,j)$ is the gain to class i of susceptibles from class j, then

$$GX(i,j) = \left[G(i) \sum_{i=1}^{N} G(i)\right] [(X(j)/N(j)]\, L(j). \tag{49}$$

The movements are made to conform to a change in behaviour that reduces the mean level of activity by a defined amount and hence losses occur mainly in the high activity classes and gains mainly in the low-activity classes; consequently the effect of this scheme is a redistribution of individuals from the high-activity classes to the low-activity classes.

(iii) *Scheme* 3

Compared with scheme 2, scheme 3 minimizes the magnitude of individual behaviour change by permitting movements only between adjacent classes (except in the event that the adjacent class cannot supply the requisite number of individuals, when recourse must be made to the subsequent class). The direction of movement is always from a higher sexual activity class

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into the adjacent lower-activity class, except when the loss required from the former exceeds the insufficiency of the latter, in which case, the excess is transferred to the adjacent higher class.

Thus we define a matrix v:

$$v(i,j) = 0,$$

$$w(i,j) = v(i-1,i) + G(i) - L(i) - \sum_{i+1}^{j-1} N_k,$$

if $w(i,j) > N_j$ then

$$v(i,j) = N_j,$$

$$v(j,i) = 0,$$

if $N_j > w(i,j) > 0$, then

$$v(i,j) = w(i,j),$$

$$v(j,i) = 0,$$

if $-L(i) < w(i,j) < 0$, then

$$v(i,j) = 0,$$

$$v(j,i) = -w(i,j),$$

if $w(i,j) < -L(i)$ then $v(i,j) = v(j,i) = 0.$

Then

$$GX(i,j) = X_j/N_j v(i,j). \tag{50}$$

(iv) *Scheme* 4

If the probability of moving between sexual activity classes depends on an individual's infection status, then it is possible to restrict behaviour changes to a particular subpopulation (i.e. susceptibles or infecteds). An extreme case would be behavioural changes being restricted to the susceptible population such that:

$$g_0(i) = \left[g'(i)\, N - \sum_{j=1}^{3} Y_j(i) \right] \Big/ X, \tag{51}$$

where no change in activity occurs within the Y_j subpopulations. In principle, it is possible to confine behavioural changes to any particular group. However, if the group is small in size, relative to the total population, then it may not be possible to create sufficient changes in the target subpopulation to satisfy the desired degree of change (i.e. the desired reduction in the overall mean and its associated variance) in the total population. In those cases, it is necessary to dispense with the notion that the total sexually active population is redistributed as $g'(i)$. Instead, it is assumed that the group in question changes to adopt the new distribution $g'(i)$ with the overall distribution changing in a manner defined by the changes in the target group and their proportional representation in the total population.

4. Model predictions

This section is divided into three parts: (*a*) a sensitivity analysis of model behaviour (particularly with respect to different parameter assignments for the set of transmission

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probabilities (the β_is) and the durations of the different infectiousness stages (the T_is); (*b*) the influence of changes in sexual behaviour and (*c*) the likely course of the epidemic in the male homosexual population of the U.K.

Certain parameters are assigned fixed values. The mean incubation period ($T = \sum T_i$) was set at eight years for sexually active adults (unless otherwise stated) in line with current estimates derived from studies of transfusion associated cases and cohorts of infected male homosexuals (Medley *et al.* 1988; Moss *et al.* 1989). With constant rates of joining and leaving the three infectious classes (Y_1, Y_2 and Y_3) and the AIDS class (A), the distribution of the incubation period is formed from the sum of three exponential distributions (see Blythe & Anderson 1988*b*). When the times (T_1, T_2 and T_3) spent in each of the three stages are 1, 4 and 3 years respectively (with $T = 8$ years), then the best fit Weibull distribution has a mean of 8.7 years, a variance of 5.92 and parameters $a_1 = 0.429$, $a_2 = 0.317$ (see Medley *et al.* 1988). The total number of male homosexuals in England and Wales was set at 500000 at the start of the epidemic (approximately 4% of the sexually active male population between the ages of 16–46 years) (see Fay *et al.* 1989). The expected duration of sexual activity in the absence of HIV infection was set at 30 years ($1/\mu$). The net immigration rate of new susceptibles into the sexually active population, Λ, was set to satisfy the constraint that $\Lambda/\mu = 500000$. Immigration into the different sexual activity classes (the Λ_is) was determined by the power law relationship between the variance (σ^2) and mean (m) of the sexual activity distribution, as defined in (3), with coefficients $a = 0.555$, $b = 3.231$ (see Blythe & Anderson 1989). The proportion of infecteds, f, who eventually develop AIDS was fixed at 1.0 for all the numerical simulations. The mean rate of sexual partner change was fixed at 8.7 different partners per year with variance defined by (3) (see Blythe & Anderson 1988*c*). Six sexual activity classes were defined with annual rates of acquiring new partners of 0–2, 2–5, 5–10, 10–20, 20–50 and 50+. For illustrative purposes, the proportions in each of these activity classes, for a mean annual rate of partner change of 8.7 and a variance of 602 (as defined by (3) with coefficients $a = 0.555$, $b = 3.231$), are depicted in figure 4.

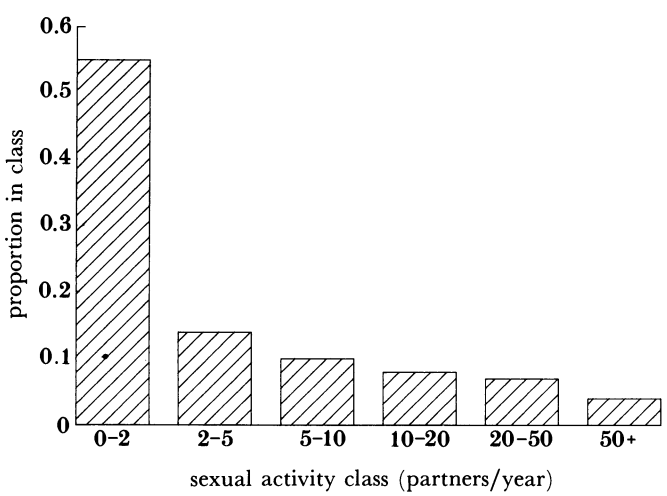

FIGURE 4. An illustration of the proportion of individuals in six sexual activity classes (0–2, 2–5, 5–10, 10–20, 20–50, 50+) (all defined as per year), for a distribution with mean, $m = 8.7$ and variance $\sigma^2 = 602$.

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(a) *Sensitivity analysis*

(i) *Variation in the magnitude and duration of infectiousness*

As described earlier the model divides infected but non-AIDS individuals into three classes, Y_1, Y_2 and Y_3, with average durations of stay in each class of T_1, T_2 and T_3 respectively. The probability of transmission (i.e. infectiousness) is asumed to be constant within each class, but to vary between classes with values, β_1, β_2 and β_3 respectively for Y_1, Y_2 and Y_3. Individuals pass sequentially from class 1, via class 2 to class 3. They enter class 1 when first infected and leave class 3 when AIDS is diagnosed. The average incubation period of the disease, T, is thus $T = T_1 + T_2 + T_3$. The average duration of infectiousness, D, is the same as T provided each $\beta_i > 0$, if not D is equal to the sum of the stage durations in which $\beta_i > 0$. The basic reproductive rate of infection R_0, is simply

$$R_0 = c[\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3], \tag{52}$$

given the assumption that the effective rate of sexual partner change, c, is the same for the three infected classes. We define the average transmission probability $\bar{\beta}$ over the average incubation period T as $\bar{\beta} = (\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3)/T$.

In this subsection we examine the influence of changes in the values of the β_is and T_is on the pattern and magnitude of the epidemic. The first set of numerical simulations of the timecourse of the epidemic in the male homosexual population (simulation runs 1–9) employ three different values of the average transmission probability, $\bar{\beta}$, namely, 0.01, 0.05 and 0.1. For each value of $\bar{\beta}$ three different simulations were done with β_1 set at 0.01, 0.03 and 0.05, respectively. The parameter β_2 was set at zero in all simulations with the average infectious period fixed at 4 years with $T_1 = 1$ year, $T_2 = 4$ years and $T_3 = 3$ years, such that the average incubation period T was fixed at 8 years. The value of β_3 was adjusted for each value of β_1 such that the desired value of $\bar{\beta}$ (i.e. 0.01, 0.05 or 0.1) was obtained. A summary of the parameter values employed for each of the nine simulations is presented in table 5. With a fixed mean (8.7 year^{-1}) and variance (602.4) for the sexual partner change distribution (defined per year), and a fixed average infectious period (4 years), the $\bar{\beta}$ values of 0.01, 0.05 and 0.1 gave R_0 values of approximately 6, 30 and 61 respectively.

We summarize the results of these simulations by presenting, in graphical form, temporal changes (for a period of 100 years from the start of the epidemic) in the total number of HIV-infected persons (number seropositive), the proportion of infected individuals in the male homosexual community, the incidence of AIDS expressed per quarter year and the total population size of the community under the impact of the AIDS epidemic (from a starting size of 500 000 males at time $t = 0$). An example of these four variables plotted against time is provided in figure 5 for simulation run number 1 (see table 5 for parameter values). This particular case is one in which the magnitude of R_0 is set at 6.13 with an average transmission probability of 0.01 over the 8-year incubation period. The graphs ((*a*)–(*d*)) in figure 5 show the overall pattern of the epidemic in a community with high average rates of sexual partner change in which no change of behaviour occurs over the 100-year timespan. Note that the epidemic rises to a peak and then settles to a stable equilibrium which, for the parameter values employed in run 1 (see table 5), occurs about 50 years after the introduction of HIV. Also note that the peak in the number or proportion infected with HIV precedes that in the incidence of the disease AIDS because of the long and variable incubation period. Population size is depressed

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TABLE 5. PARAMETER VALUES OF SIMULATION RUNS 1–9

simulation number	average incubation period T/years	average period/years	infectious period: transmission probabilities β	β_1	β_2	β_3	stage duration/years T_1	T_2	T_3	*basic reproductive rate R_0	sexual activity classes (mean partner change in class/year) 0–2 (0.55) 1	2–5 (0.14) 2	5–10 (0.1) 3	10–20 (0.08) 4	20–50 (0.07) 5	50+ (0.04) 6	*overall distribution mean m	variance σ_2
1	8	4	0.01	0.01	0	0.023	1	4	3	6.23	0.45	3.21	7.03	13.85	43.22	81.31	8.7	602
2	8	4	0.01	0.03	0	0.017	1	4	3	6.23	0.45	3.21	7.03	13.84	43.63	81.23	8.7	602
3	8	4	0.01	0.05	0	0.01	1	4	3	6.23	0.45	3.20	7.02	13.83	43.97	80.28	8.7	602
4	8	4	0.05	0.01	0	0.13	1	4	3	31.15	0.32	3.10	6.91	13.68	43.75	80.35	8.7	602
5	8	4	0.05	0.03	0	0.123	1	4	3	31.15	0.32	3.10	6.91	13.68	43.75	80.35	8.7	602
6	8	4	0.05	0.05	0	0.117	1	4	3	31.15	0.32	3.10	6.91	13.68	43.75	80.35	8.7	602
7	8	4	0.1	0.01	0	0.263	1	4	3	62.35	0.25	3.08	6.89	13.67	44.3	80.28	8.7	602
8	8	4	0.1	0.03	0	0.257	1	4	3	62.35	0.25	3.08	6.89	13.67	44.3	80.28	8.7	602
9	8	4	0.1	0.05	0	0.25	1	4	3	62.35	0.25	3.08	6.89	13.67	44.3	80.28	8.7	602

* Approximate values.

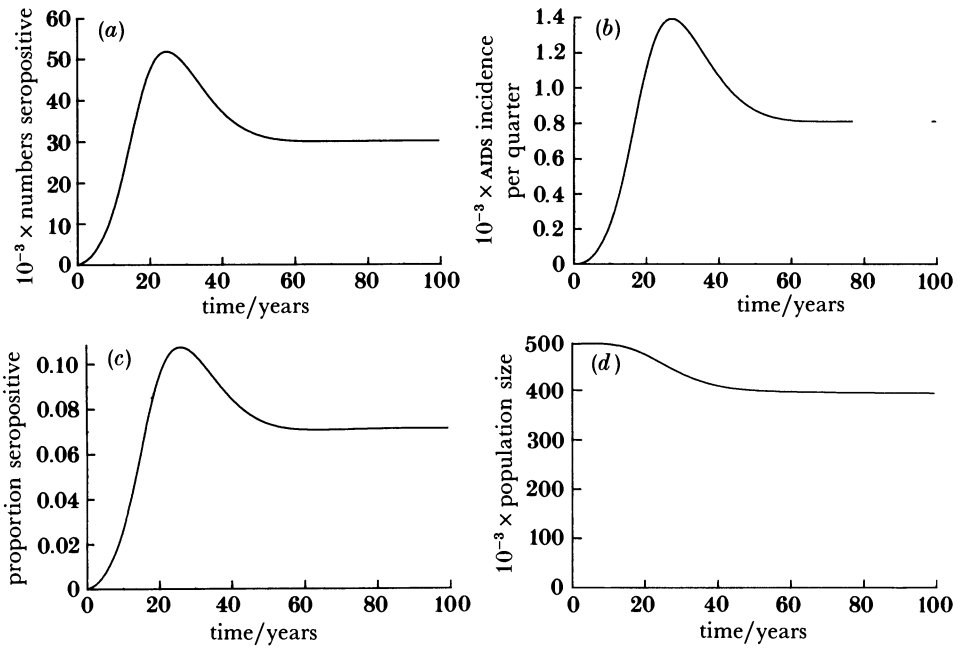

FIGURE 5. Simulation run 1. Temporal changes in (*a*) the number seropositive, (*b*) the quarter year incidence of AIDS, (*c*) the proportion seropositive in the population (= infected with HIV) and (*d*) the size of the population, as predicted by the model with parameter assignments as defined in run 1 in table 5.

from the disease-free level as a result of the mortality induced by AIDS, despite a constant rate of recruitment of new susceptible into the sexually active population.

A clearer picture of some of the details of the course of the simulated epidemic is displayed in figure 5 for run number 1. Graph (*a*) records changes through time in the numbers of individuals in each of the sexual activity classes (1–6) that record the rate of acquiring new sexual partners per year (see table 5). Snapshots at six time intervals are recorded for each sexual activity class. For a given class the left-hand bar denotes time zero and each subsequent bar denotes the numbers in that class at 10-year intervals (i.e. 0, 10, 20, 30, 40, 50 years). Note that mortality due to AIDS has the greatest proportional impact on those in the highest sexual activity class (6). After 50 years of the epidemic the numbers in this class are less than 30 % of the size of the group at time $t = 0$. Also note that for the assigned parameter values for run number 1 the epidemic has little impact on activity class 1. Those with high rates of sexual partner change are both more likely to acquire infection and die from AIDS and to transmit it to others. In the early stages of the epidemic 71 % of the total sexual partner changes in a given year are made by individuals in activity classes 5 and 6 who represent only 11 % of the total population.

Graph (*b*) in figure 6 records changes through time in the proportion infected with HIV in each of the six sexual activity classes. Note that the increase in seropositivity is very rapid in the high activity class but very slow in the low activity class. In class 6 seropositivity attains a maximum value of around 80 % whereas in class 1 seropositivity only rises to a few percent.

The shape and overall magnitude of the epidemic, the time interval between the start of the epidemic and the point of maximum disease incidence and the equilibrium incidence of disease and infection, depend on the magnitude of the basic reproductive rate R_0 and the values of its

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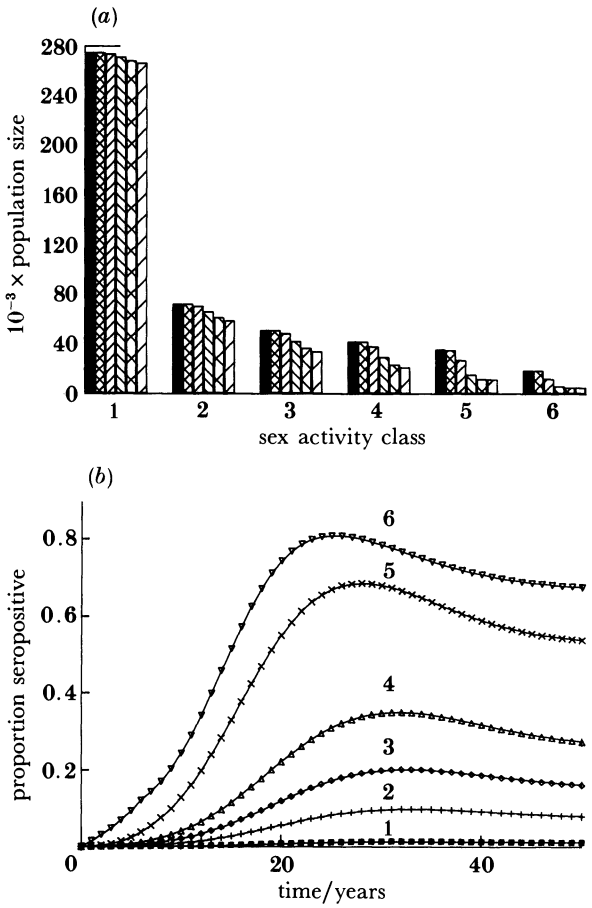

FIGURE 6. Simulation run 1. (*a*) Changes through time in the number of individuals in each of the six sexual activity classes (see main text). Snapshots at six time intervals are recorded for each activity class where the left-hand bar denotes time $t = 0$, and each subsequent bar denotes the numbers in that class at 10-year intervals, i.e. $t = 10, 20, 30, 40, 50$. (*b*) Changes through time in the proportion infected with HIV in each of the six activity classes.

component parameters (i.e. the β_is, T_is and c). Figure 7 illustrates the influence of the magnitude of the transmission probability during the first phase of infectiousness, β_1, on the shape of the epidemic, as represented by the incidence of AIDS (0.25 per year), for various values of R_0. In graphs (*a*), (*b*) and (*c*) the value of β_1 is set at 0.01, 0.03 and 0.05 respectively. In each graph the value of β_3 ($\beta_2 = 0$) is varied to give three different values of R_0, respectively 6 ($\bar{\beta} = 0.01$), 31 ($\bar{\beta} = 0.05$) and 62 ($\bar{\beta} = 0.1$). Note that the value of β_1 has a strong influence on the rate of increase in the epidemic over its early stages, and hence on the timing of peak incidence. High values of β_1 lead to rapid rises in incidence and an early peak, while low values lead to a less rapid rise and a later peak. The value of R_0, however, determines the overall magnitude of the epidemic and the level of the endemic equilibrium state (compare each of the three trajectories within each graph, see table 5 for parameter values). High values of R_0 result in rapid growth of the epidemic and it is more 'peaked' in character by comparison with that generated by lower values (figure 7). A clearer picture of the influence of the magnitude of β_1 on the rate of growth of the epidemic in its early phase is provided in figure 8 where the value of R_0 is held constant in each individual graph (6 in (*a*), 31 in (*b*) and 62 in (*c*)) and β_1 is varied

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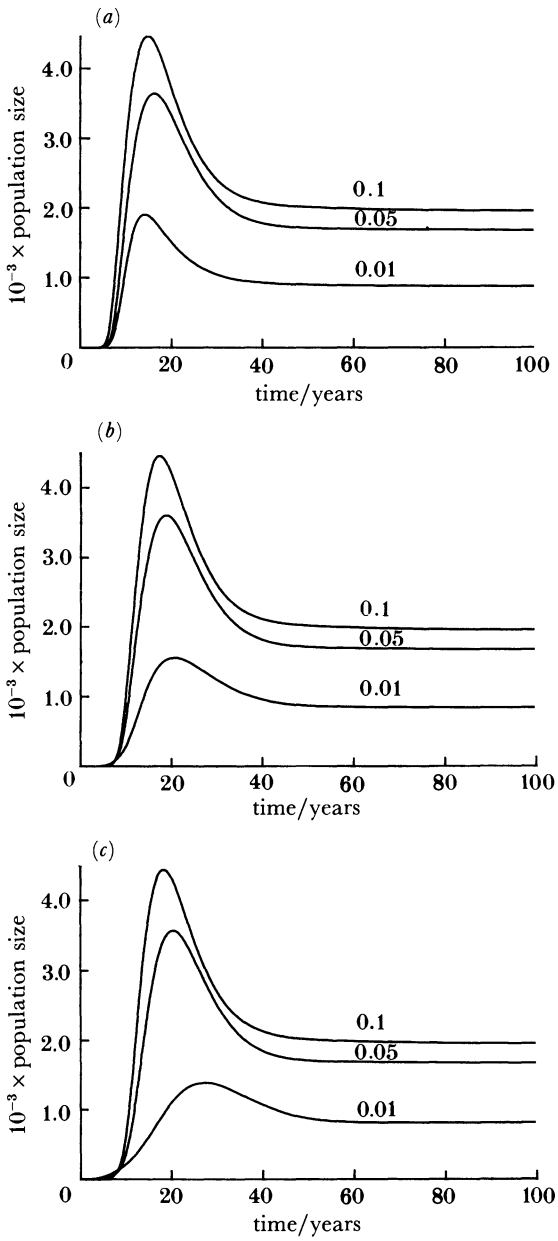

FIGURE 7. Temporal changes in the incidence of AIDS (defined per quarter year). In graph (*a*) the value of β_1 was held constant at 0.05 while the value of β_3 was varied to give β values of 0.1, 0.05 and 0.01 with $T_1 = 1$, $T_2 = 4$, $T_3 = 3$ (simulation runs 3, 6 and 9, in table 5). (*b*) Similar to (*a*) except that the value of β_1 was fixed at 0.03 (simulation runs 2, 5 and 8 in table 5).

from 0.01 to 0.05. The magnitude of β_1 in the first phase of infectiousness determines the rapidity with which the epidemic spreads, while the value of β_3 in the second phase of infectiousness helps to determine the magnitude of R_0, and hence the overall size of the epidemic (Anderson 1988*a*; May & Anderson 1988; Anderson & May 1988).

The rapidity of the rise in the incidence of AIDS reflects the rapid rise of the proportion infected with HIV in each of the six sexual activity classes. This is shown in figures 6*b* and 9. In both graphs the magnitude of R_0 was set at 6 but in figure 6*b*, β_1 was set at 0.01, and in

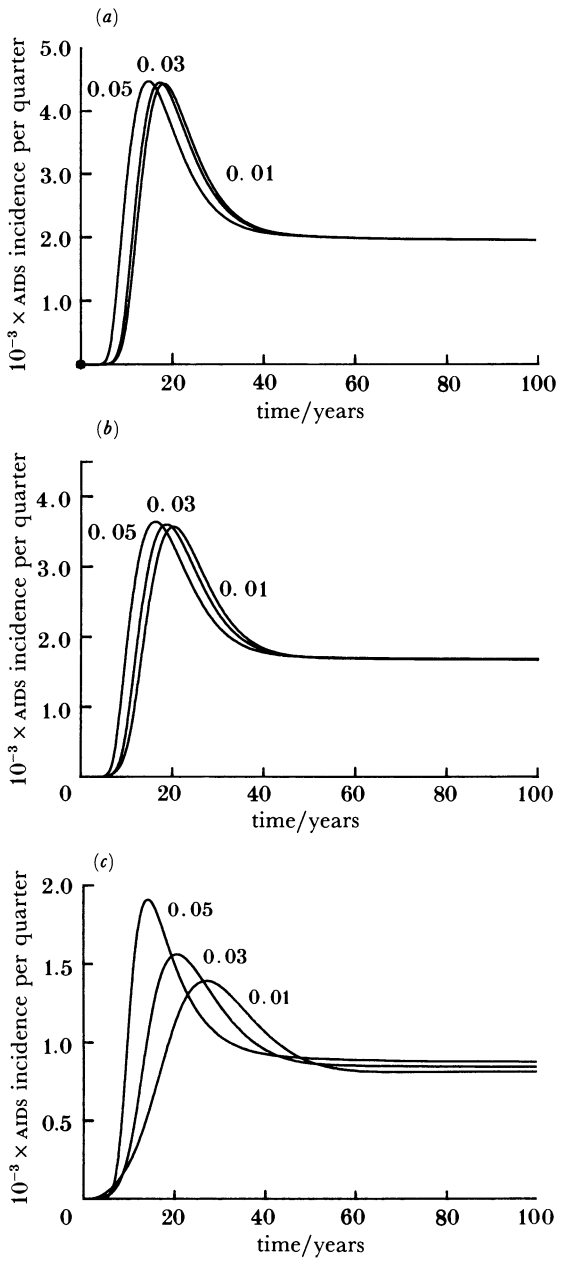

FIGURE 8. Temporal changes in the incidence of AIDS (defined per quarter year). In graph (*a*) the value of $\bar{\beta}$ was held constant at 0.1 while the value of β_1 was varied in 3 separate simulations with values of 0.01, 0.03 and 0.05 ($R_0 = 62.3$) (simulation runs 7, 8 and 9 in table 5). (*b*) Similar to (*a*) except that the value of $\bar{\beta}$ was fixed at 0.05 ($R_0 = 31.15$) (simulation runs 4, 5 and 6 in table 5).

figure 9, β_1 was set at 0.05 (run 3, see table 5). Note that with a high value of β_1 (0.05) for a fixed value of R_0 seroprevalence rises rapidly such that it attains a maximum value in the highest sexual activity class (6) 10 years after the start of the epidemic (figure 9). By contrast, for the low value of β_1 (0.01) the maximum proportion infected in class 6 occurs 20 years after the start of the epidemic (figure 6*b*). For higher values of R_0 (i.e. 31, with $\bar{\beta} = 0.05$, $\beta_1 = \beta_3 = 0.1$) the rapidity of rise in seroprevalence through time is even more marked (figure 10). The

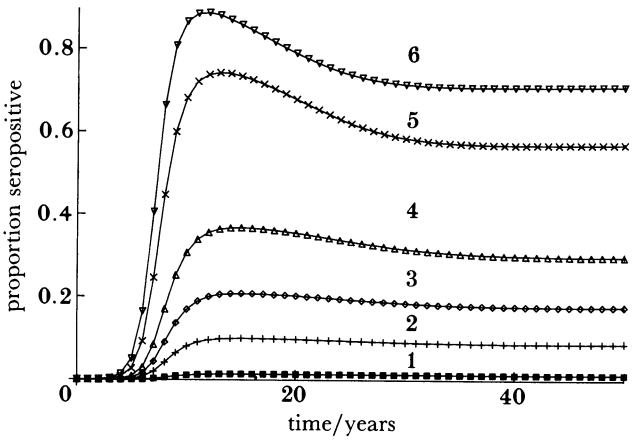

FIGURE 9. Temporal changes in the proportion infected with HIV (seropositive) in the six different sexual activity classes (1–6, see text). Parameter values were set as defined for simulation number 3 in table 5.

example recorded in figure 10 shows that HIV prevalence in the highest sexual activity class (6) again exceeds 90% in around five years from the start of the epidemic (graph (*b*)) even though the prevalence in the total population only attains a maximum level of around 40% 10–12 years after the start of the epidemic. This pattern is similar to that observed in male homosexuals attending sexually transmitted disease clinics in San Francisco in the U.S.A. in the period 1980–1985. For those with very high rates of sexual partner change seroprevalence rose from a few percent to around 90% over the five-year period (Curran *et al.* 1988; CDC 1985). Broader-based surveys of prevalence of HIV infection in male homosexual communities in the U.S.A. in 1987–1988 suggest average prevalence levels of around 36% (Curran *et al.* 1988; CDC 1988) about 10–12 years after the emergence of the epidemic in this risk group.

Also of interest in the simulated trajectories of the rise in the proportion of the population infected with HIV is the rate of change in the numbers or proportions infected during the course of the epidemic. As illustrated in figure 11 for simulation runs 4, 5 and 6, the rate of change in the numbers infected per quarter year rises rapidly in the early phase of the epidemic, declines in the middle phase, becomes negative just after the peak of the epidemic before settling to zero as the equilibrium state is reached. The nonlinear character of these changes is of particular importance. Note that before the peak in the numbers infected the rate of change begins to decrease quite markedly even in the absence of changes in sexual behaviour. This is a consequence of 'saturation' effects in the high sexual activity classes where, because of the high proportion infected, both partners in many sexual liaisons are already infected.

Observed longitudinal trends in the prevalence of HIV infection in populations of male homosexuals depend to some extent on the method employed to sample the population (Winkelstein *et al.* 1987). The study referred to above in San Francisco was based on homosexual/bisexual men recruited from sexually transmitted disease clinics in 1978 for ongoing studies of hepatitis B virus. A selected sample from this population who consented to blood tests in each year from 1978 to 1986 (i.e. a cohort study) revealed the trend recorded in figure 12*a* (sample size 283 in 1978; CDC 1988). Prevalence again rose rapidly over the time interval 1980–1985, and plateaued at around 60% by 1986. By way of a contrast, a non-cohort study of homosexual/bisexual men attending a sexually transmitted disease clinic in London showed a rapid rise in seroprevalence from 1981 to 1983 but thereafter the percentage of

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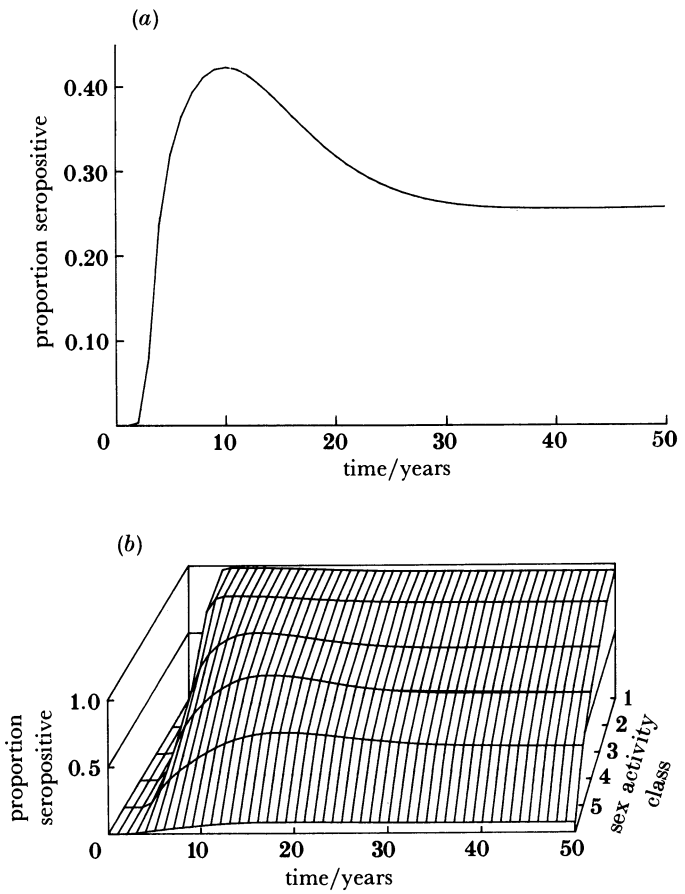

FIGURE 10. Temporal changes in the proportion infected with HIV (seropositive) (*a*) in the total population and (*b*) in the six different sexual activity classes (1–6). Parameter values: $R_0 = 31$, $\bar{\beta} = 0.05$, $\beta_1 = \beta_3 = 0.1$, $T_1 = 1$, $T_2 = 4$, $T_3 = 3$.

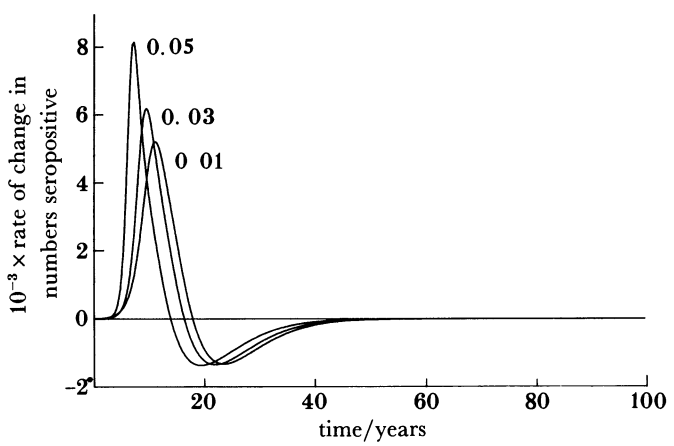

FIGURE 11. Temporal changes in the rate of change (per quarter year) in the numbers seropositive for HIV. Trends through time are recorded for simulation runs 4, 5 and 6 in table 5 (β values of 0.01, 0.03 and 0.05).

infected attendees at the clinic plateaued at around 25% (Carne *et al.* 1987; HMSO 1988) (figure 12*b*). In both cases the attained of a plateau was in part due to saturation effects and in part because of changes in sexual behaviour. This latter explanation is thought to be a major factor contributing to the relative low percentage infected in the London study, in comparison with that in San Francisco. The epidemic started later in London and hence a knowledge amongst homosexuals of what was occurring in the U.S.A. is believed to have stimulated changes in sexual behaviour at an earlier stage in the epidemic by comparison with that in San Francisco.

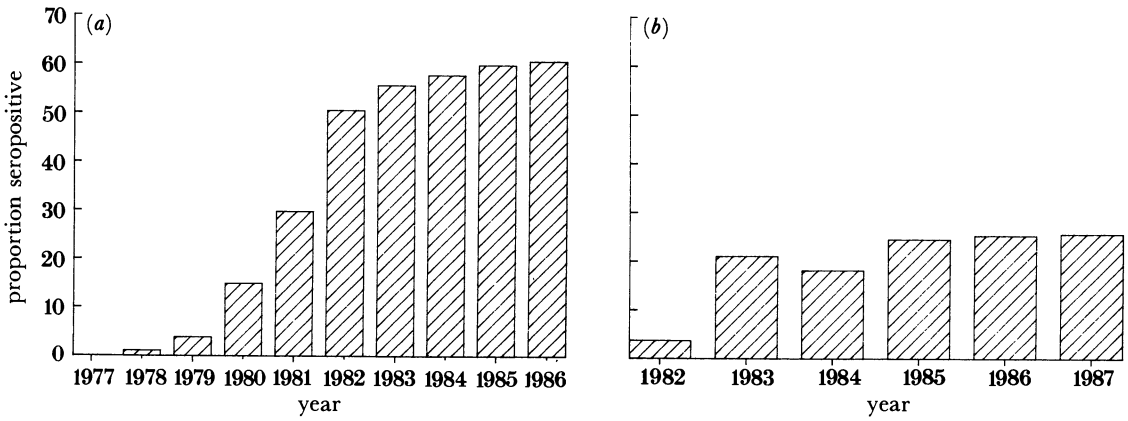

FIGURE 12. Temporal trends in the proportion seropositive for HIV recorded in two studies of groups of male homosexuals attending sexually transmitted disease clinics (*a*) in San Francisco, U.S.A. and (*b*) in London, U.K. The data in (*a*) are from table 12 in CDC (1987) and those in (*b*) are from Carne *et al.* (1987), table A 3.1 in HMSO (1988) and from Loveday *et al.* (1989). See source references for details of the study populations.

(ii) *Variation in the length of the non-infectious stage,* T_2

Changes in the average duration of the non-infectious period, T_2, between the two peaks in infectiousness can have a significant influence on the pattern of the epidemic. This point is illustrated in figure 13 where the magnitude of T_2 is varied (with values of 3, 4, 5 and 10 years) while β_1 and β_3 are held constant with values of 0.03 and 0.0167, respectively. Keeping β_1 and β_3 constant with fixed values of T_1 (1 year) and T_3 (3 years) constrains the magnitude of R_0 to the constant value of 6.2, irrespective of the value of T_2 (provided $\beta_2 = 0$). The epidemic is smaller in magnitude, is less 'peaked' in character and the equilibrium incidence of AIDS is lower when the value of T_2 is high, compared with lower values. This is in part a consequence of the long drawn out nature of the epidemic such that with long non-infectious periods a significant fraction of those infected in the early stages of the spread of infection (i.e. those in the high sexual activity classes) will have either died from natural causes or ceased sexual activity before progressing to the second phase of infectiousness.

(*b*) *Changes in sexual behaviour during the course of the epidemic*

Beneficial changes in sexual behaviour may occur via a reduction in the rate of acquiring new sexual partners (i.e. a change in the values of m and σ^2) or via an increase in the frequency of adoption of what are termed 'safer sex practices' (changes in the β_is). The latter include the use of condoms and a reduction in high risk behaviours such as anal sex. Such changes can be

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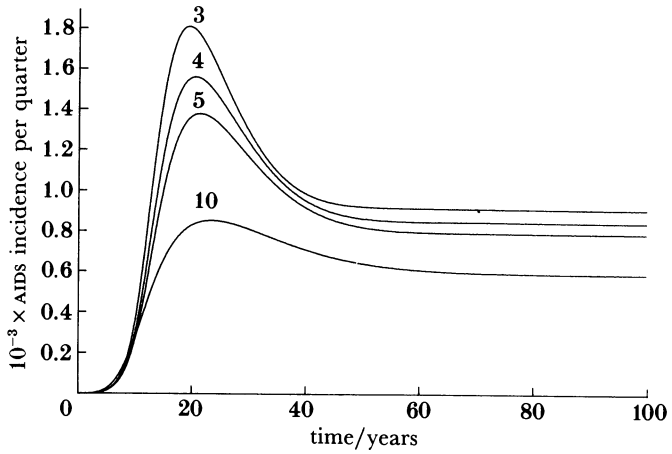

FIGURE 13. The effect of varying the duration of the non-infectious period (of length T_2 years) between the two periods of infectiousness on the predicted temporal trend in the incidence of AIDS (per quarter year). Four simulations are plotted with the value of R_0 held constant at 6.2 and the value of T_2 varied in the separate simulations to 3, 4, 5 and 10 years ($T_1 = 1$ year, $T_2 = 3$ years).

mirrored in the model in various ways. As described in the methods section of the paper we examine three different methods of representing changes in behaviour. These are: (i) reductions in the average rate of sexual partner change (m) among susceptible recruits to the sexually active population, via education at schools; (ii) reduction in the magnitudes of the transmission probabilities (the β_is) to mirror the adoption of 'safer sex practices' and (iii) reduction in the average rate of sexual partner change among recruits and the sexually active population via education in the general population and at schools.

(i) *Change in the rate of acquisition of new sexual partners among susceptible recruits to the sexually active population*

Changes in the behaviour of new recruits to the sexually active population (the Λ_is) were simulated in two different ways. In the first type of simulation, the mean rate of sexual partner change of the susceptible, recruits was reduced by a defined amount from 1983 onwards (the epidemic was assumed to have started in 1979, hence $t = 0$ in all simulations denotes 1979, $t = 10$ denotes 1989, etc.). Those recruited before 1983 and those sexually active before this date were assumed not to change their behaviour. In five different simulations, the mean rate of partner change, m (with the variance σ^2 calculated from the power low relation, see figure 1), was changed from 8.7 per year in the interval 1979–1983, to 6, 5, 4, 3 and 2 (all per year) from the beginning of 1983 onwards, respectively. The impact of such changes on the quarter-yearly incidence of AIDS is depicted in figure 14*a* for each of the five simulations. For comparison the incidence of AIDS in the absence of any behavioural change is also plotted against time. The parameter set used to generate these trajectories, aside from the differing assumptions concerning the mean rate of sexual partner change, m, were as defined for simulation run number 2.

In the second type of simulation the mean rate of acquiring sexual partners by new susceptible recruits to the sexually active population was reduced in a stepwise manner from 8.7 per year in 1979–1983, by one unit in each successive year up to 1988 (i.e. mean rates of sexual partner change of 7 per year in 1983, 6 per year in 1984, 5 per year in 1985, 4 per year

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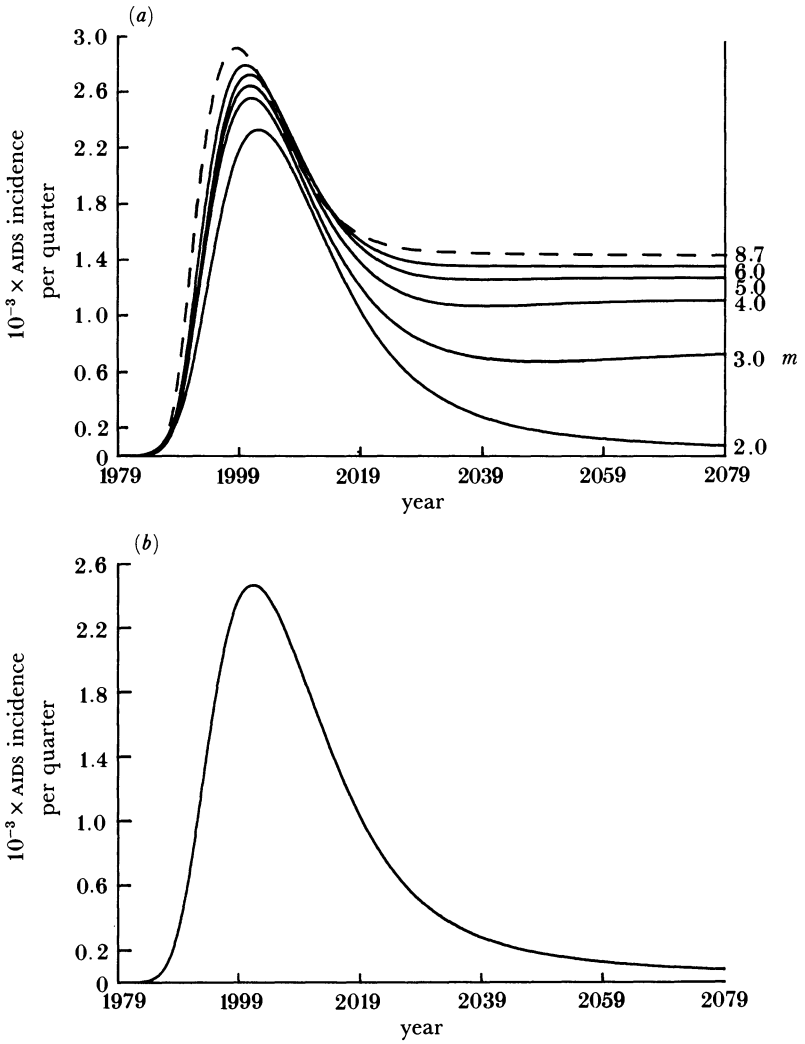

FIGURE 14. The impact of changes in sexual behaviour (reductions in the rate of acquisition of new sexual partners) among susceptible recruits to the sexually active population (with no change in behaviour among individuals in the sexually active population at the time point when changes in behaviour were initiated amongst the recruits). In (*a*) five simulations are recorded in which the mean rate of sexual partner change per year of recruits was reduced in 1983 (and thereafter) to 6.0, 5.0, 4.0, 3.0 and 2.0. The simulation for no change ($m = 8.7$ per year) is also plotted to provide a point of reference. Parameter values were set as defined by simulation run number 2 in table 5. In (*b*) the parameter values were as defined for (*a*) except that the mean rate of sexual partner change among new recruits was reduced in a step-wise manner from 8.7 per year in 1979–1983 by one unit in each success year up to 1988 (i.e. mean rates of 7 per year in 1983, 6 per year in 1984, 5 per year in 1985, 4 per year in 1986, 3 per year in 1987 and 2 per year from 1988 onwards).

in 1986, 3 per year in 1987 and 2 per year from 1988 onwards). Temporal changes in the quarter yearly incidence of AIDS under this pattern of change in behaviour is presented in figure 14*b*.

An assessment of the patterns recorded in figure 14 suggests that if the mean rate of partner change of the recruits is lowered significantly (i.e. from 8.7–2.0 in 1983) after the initial epidemic, the infection will die out in the population over a timespan of many decades (figure 14*a*). If the reduction is insufficient to reduce the overall basic reproductive rate, R_0, below unity in value, the behavioural change among new recruits will simply act to lower the level

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of the endemic equilibrium incidence of disease. Overall, the reductions in the total number of AIDS cases induced by these changes are limited in nature, by comparison with the trajectory displayed for the simulation in which no changes occur (figure 14*a*). In other words, changes in the behaviour of the new recruits to the sexually active population are insufficient by themselves to have a marked impact on the initial epidemic of AIDS, before the attainment of the endemic equilibrium state. The reason for this is that, with a life expectancy of 30 years of sexual activity (in the absence of HIV infection), it takes a long period of time before the recruits (with their changed sexual behaviour) begin to constitute a majority of the sexually active population. Education must therefore be aimed at adults (the sexually active population) as well as teenagers (future recruits).

(ii) *Reductions in the transmission probability by the adoption of safe-sex practices*

Changes in behaviour via the adoption of 'safer sex' practices act to decrease the probability that the infection is transmitted during sexual contact between susceptible and infectious individuals. We simulated two different scenarios. In both cases changes in behaviour were assumed to reduce the values of the β_is by a similar amount (i.e. the overall value $\bar{\beta}$ was reduced by a defined amount). In the first type of scenario $\bar{\beta}$ was reduced to 0.75, 0.5 or 0.25 of its original value in different simulations. In separate simulations these changes were assumed to occur in 1983, 1986 and 1989. The results, expressed as temporal changes in the cumulative number of AIDS cases, are presented in figure 15. Three main points are illustrated by these numerical experiments. First, the proportional reduction in the total number of cases of AIDS is not equal to the proportional reduction in the magnitude of $\bar{\beta}$. Proportionally greater reductions in the cumulative number of cases of AIDS result from the larger reductions in $\bar{\beta}$ (i.e. a reduction of 75% for example; figure 15). The second feature is the great benefit that accrues from changes in behaviour early in the course of the epidemic. Changes in 1983, for example, have a much more substantial impact than changes initiated in 1986 (figure 15*a*, *b*). The final point of importance is that none of the proportional reductions in $\bar{\beta}$ are sufficient to reduce the magnitude of R_0 below unity. As such the infection persists and the cumulative number of cases of AIDS continues to grow over a timespan of many decades. For a large reduction in $\bar{\beta}$ the epidemic moves on an extremely long timescale.

The second type of simulation involved the cessation of transmission by setting $\bar{\beta} = 0$ at different time points in the course of the epidemic. Behavioural changes that blocked transmission were induced in separate simulations each year (1983–1989). The trajectories through time of the quarter year incidence of AIDS, and the total number of infected individuals, are recorded for each simulation in figure 16*a*, *b*. The principle point illustrated by this simulation is the large number of AIDS cases that occur after the cessation of transmission as a direct result of the long and variable incubation period of the disease. Note that even if all transmission ceased in 1987, significant numbers of AIDS cases would still be reported over the subsequent 20-year period.

(iii) *Reductions in the rate of sexual partner change*

The third method of simulating changes in behaviour centres on changes in the probability distribution of the rate of acquiring new sexual partners per unit of time. Simulating such changes raises many complications in model formulation and requires careful definition of exactly how such changes are induced and who they influence. As described in the Methods

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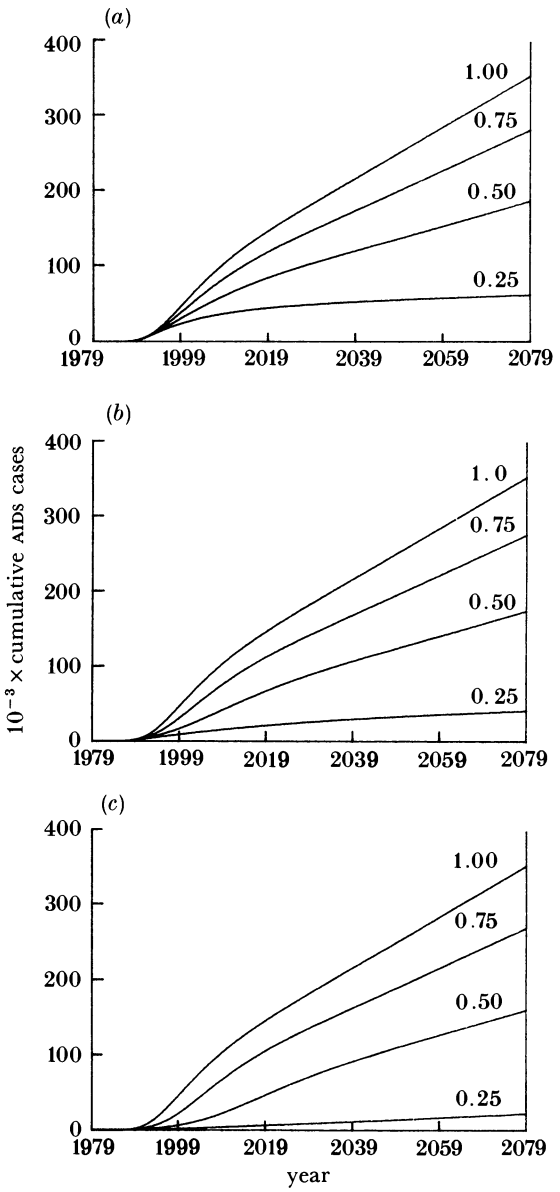

FIGURE 15. The influence of changes in the probability of transmission (β) induced by the adoption of safer-sex practices on the cumulative number of AIDS cases over a 100-year time span. The magnitude of $\bar{\beta}$ was reduced by 0.75, 0.5 and 0.25 of its pristine value in (*a*) 1989, (*b*) 1986 and (*c*) 1983 in separate simulations. Before the change the parameter values were as defined for simulation run number 2 in table 5. For a 25% reduction (in (*a*) 1989, (*b*) 1986 or (*c*) 1983) the parameter values were set at $\beta_1 = 0.0225$, $\beta_3 = 0.0125$, $R_0 = 4.6$. For a 50% reduction the values were set at $\beta_1 = 0.015$, $\beta_3 = 0.00835$, $R_0 = 3$ and for a 75% reduction the values were $\beta_1 = 0.0075$, $\beta_3 = 0.0042$, $R_0 = 1.5$. In all cases $T_1 = 1$, $T_2 = 4$, $T_3 = 3$ (years).

section we consider four schemes of behavioural change and examine their impact on the number of infected and the incidence of AIDS via simulations over a 100-year times period. For comparative purposes graphs denoting the outputs of separation simulation experiments also record temporal trends in the number infected or the incidence of AIDS under assumptions of no change and of changes restricted to the susceptible recruits to the active population.

The manner in which changes are induced (i.e. schemes 1–4) has a significant impact on the

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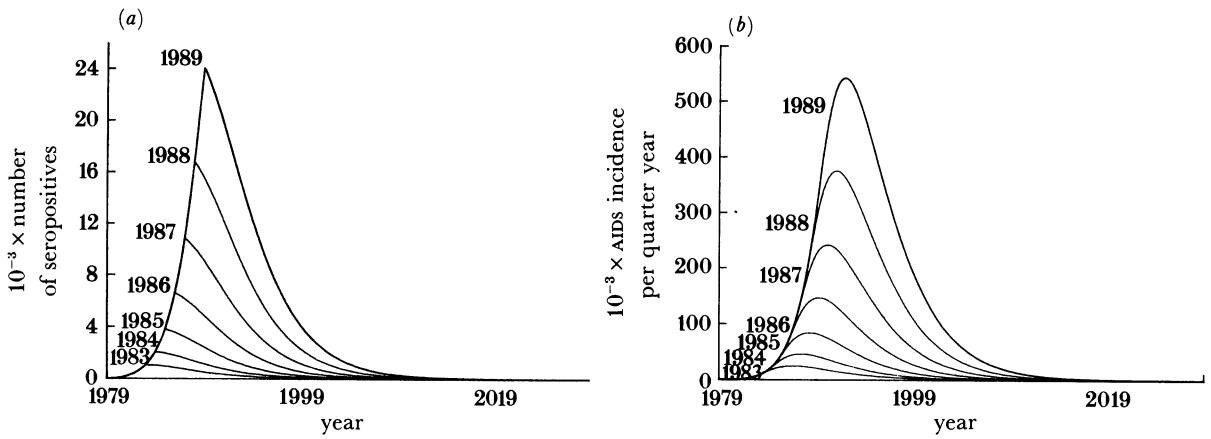

FIGURE 16. The influence of a cessation of transmission ($\bar{\beta} = 0$) induced at different time points (1983–1989) on the number of infected individuals in (*a*) (seropositives) and the incidence of AIDS per quarter year (*b*).

predicted temporal trends. In figure 17, for example, changes in the numbers infected are recorded for schemes 1–4 under the assumption that the mean rate of sexual partner change in the total population altered from 8.7 per year in the interval 1979 up to the end of 1988, to 5.0 per year from the beginning of 1989 onwards. In this particular example the changes made under schemes 1–4 satisfied the constraints defined in (47) and (48) (i.e. the mean rate of sexual partner change in the total population was 5.0 from the beginning of 1989 onwards). Under scheme 4, changes in behaviour were only induced in the susceptible population in 1989 and subsequent recruits who joined this subpopulation over the interval 1989–2079. Note that all the simulations converge to the same endemic state after many decades (except the situation where no behavioural change occurred). Also note that the greatest impact immediately

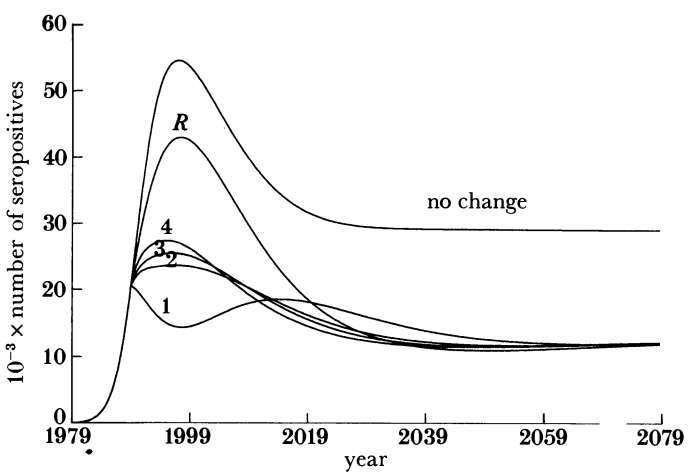

FIGURE 17. The influence of changes in the rate of sexual partner change in the sexually active population on the number of infecteds (seropositives). Various assumptions were made in different simulations concerning how the changes occurred and who they affected. As described in the text, four different schemes were simulated (labelled 1–4). A reduction in the mean rate of sexual partner change from 8.7 to 5.0 per year was made at the beginning of 1989 (parameter set as defined by simulation run number 2 in table 5). For reference the simulated trajectories for no change (labelled no change) and for changes in behaviour only among new recruits to the sexually active population (labelled *R*) are recorded in the graph.

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following the instigation of the changes results from scheme 1 in which individuals move to new activity classes independent of infection status and past sexual activity. This pattern of 'random' movement induces the greatest change amongst the infected group since, on average, individuals in this subpopulation have higher rates of sexual partner change before the introduction of changes in behaviour. The induced change in behaviour via random reassortment from all classes of activity has the greatest impact on the average level of activity in this group and, as such, has the greatest immediate impact on the net rate of transmission within the population. The pattern of change in the incidence of AIDS under these assumptions are recorded in figure 18*a*.

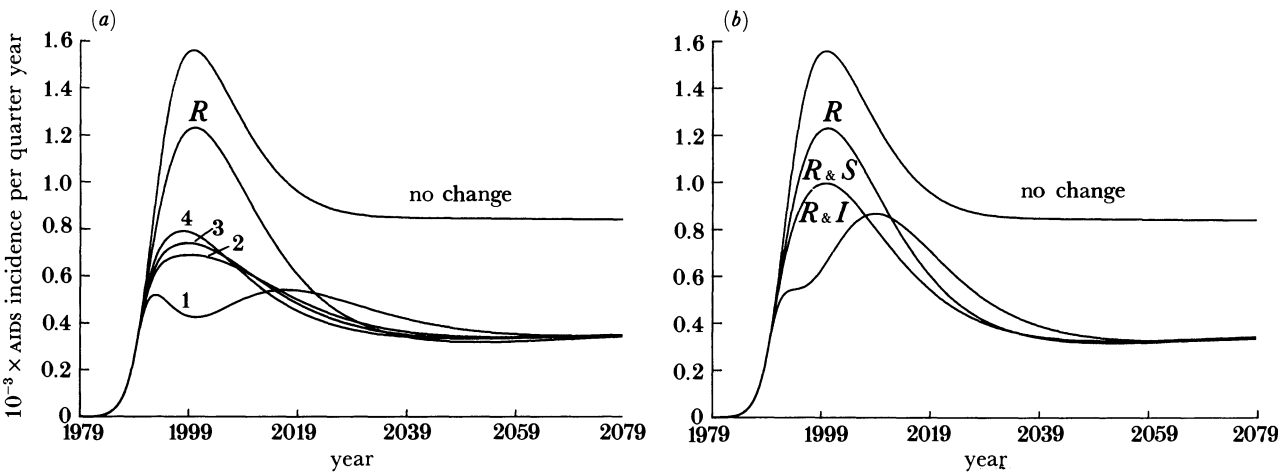

FIGURE 18. (*a*) The influence of a change in sexual behaviour among the sexually active population on the incidence of AIDS per quarter year. The different simulations and parameter sets are as defined in the legend to figure 17. (*b*) The influence of a change in behaviour (initiated in 1989) in particular infection subpopulations on the quarter yearly incidence of AIDS. As described in the text, the assumption made were as defined in scheme 4 in the methods section, where changes in behaviour are induced in particular infection subpopulations (R = susceptible recruits, S = sexually active susceptibles and I = sexually active infecteds (but not AIDS patients)), without making adjustments to satisfy the constraints defined in equations (49) and (48). At the beginning of 1989 the mean rate of partner change in a defined infection subpopulation was reduced from 8.7 to 5.0 per year (with associated changes in the variance as defined in equation (4)). The parameter set employed was as defined for simulation run number 2 in table 5.

The influence of a different pattern of changes in behaviour is recorded in figure 19 for scheme 4. In this example, changes in behaviour are induced in particular infection subpopulations only, without making adjustments to satisfy the constraints defined in (49) and (48) in the population as a whole (in contrast with scheme 4 in figure 17). The greatest impact is apparent when changes occur in the new recruits and the infected population (the mean rate of sexual partner change in the target subpopulation was reduced from 8.7 to 5.0 at the beginning of 1989). Changes in the numbers infected are recorded in figure 19 and changes in the incidence of AIDS are recorded in figure 18*b*.

The general point illustrated by these different simulations is the importance, to the pattern of the epidemic, both of the manner in which behaviour changes and of the precise group influenced by such changes. Of the four schemes considered above, we believe that scheme 3 is more likely to reflect reality. This scheme is the most 'conservative' in the sense that changes are assumed to occur between adjacent classes of sexual activity. In other words, an overall

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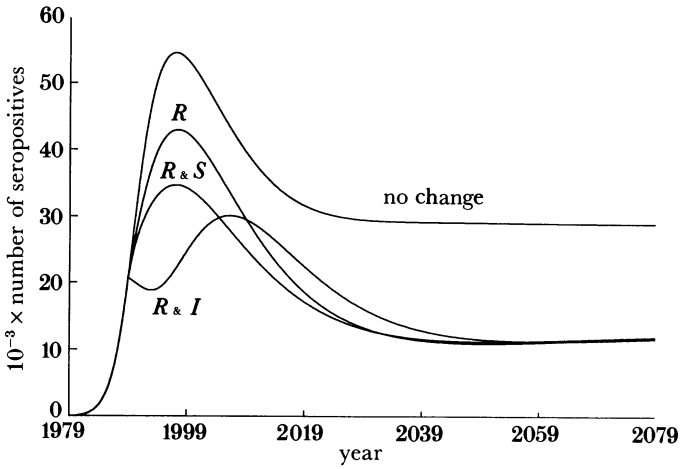

FIGURE 19. Identical to the simulations recorded in figure 18*b* but denoting the simulated changes through time in the number seropositive (parameter set number 2 in table 5).

reduction in the mean level of activity in the population occurs via individuals moving from their current activity class to the next lowest activity class.

So far we have only considered a single change in behaviour (a reduction in the mean from 8.7 to 5.0) occurring at one point in time, i.e. 1989). The timing of the change in behaviour, relative to the starting point of the epidemic (1979), has a significant impact on the overall magnitude of the epidemic. Adopting scheme 3, figure 20 records changes in the numbers infected (graph (*a*)) and the incidence of AIDS (graph (*b*)) for simulations in which the change in behaviour occurred at different time points, namely in 1983, 1985, 1987 and 1989. Note that all simulation converge after many decades on the same equilibrium point. Changes in behaviour early on in the epidemic (i.e. 1983) have a major impact on the number infected and the incidence of AIDS over the first two decades.

Substantial changes in sexual behaviour among the male homosexual community in England and Wales have occurred since the start of the AIDS epidemic, as illustrated in figure 2. The precise timing of these changes and their magnitudes is difficult to ascertain from current data. Two recent studies of HIV incidence and sexual behaviour in male homosexual communities in London and Amsterdam suggest that major changes in the rate of infection and pattern of behaviour occurred in these two populations around 1985–1986 (Evans *et al.* 1989; van Griesven *et al.* 1989). For example in the study of van Griesven *et al.* (1989) in Amsterdam, the incidence of HIV infection (defined as the percentage of the study cohort who acquired HIV per annum) increased sharply over the interval 1981–1984 and then fell markedly in the period 1985–1987 (figure 21). As illustrated by our numerical simulations (see figure 11) a rapid rise followed by a rapid fall in the incidence of new infections is to be expected, even in the absence of behavioural changes, because of saturation effects in the high activity classes. The studies in London and Amsterdam were to some extent focused on high activity classes (i.e. attendees at sexually transmitted disease clinics), but the behavioural data collected in these two surveys did suggest that significant changes in behaviour, both with respect to the frequency of high-risk activities plus rates of sexual partner change, did occur during the period 1986–1987. Unlike our earlier simulations, such changes have presumably occurred in a continuous manner since

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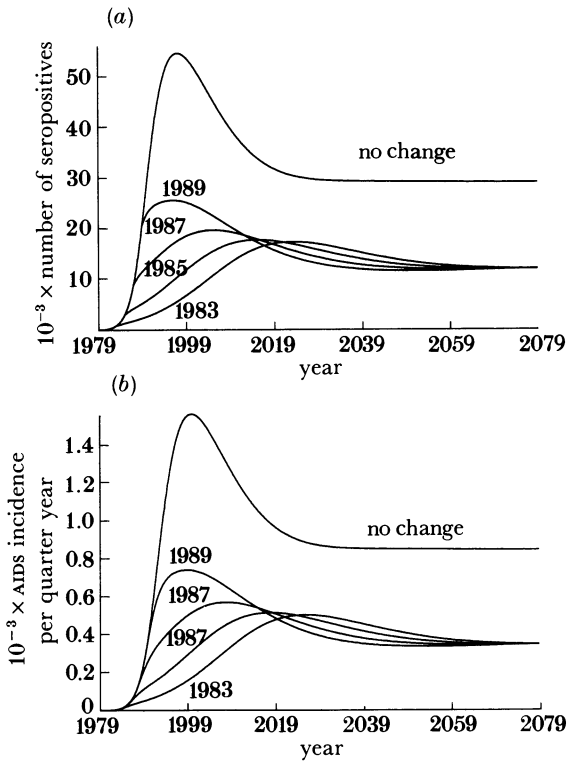

FIGURE 20. The influence of the timing of a change in sexual behaviour on temporal trends in the quarter yearly incidence of AIDS in (*b*) and the number infected (seropositive) in (*a*). In separate simulations a single alteration in behaviour, in which the mean rate of sexual partner change was reduced from 8.7 to 5.0 per year (with an associated change in the variance) under the assumptions of scheme 3, was introduced either in 1983, 1985, 1987 or 1989. The parameter set was as defined for simulation run number 2 in table 5.

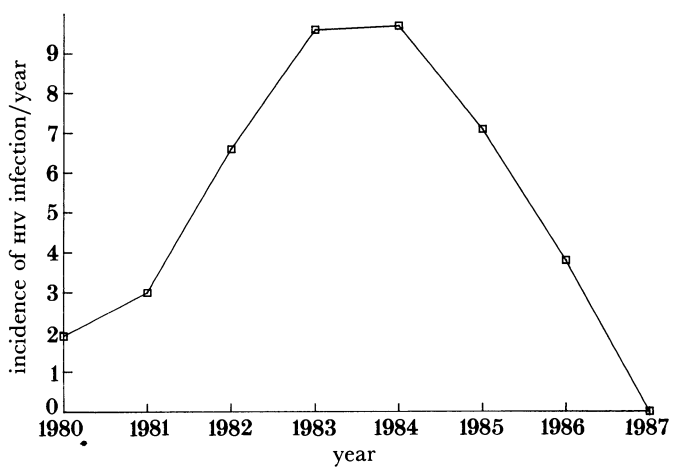

FIGURE 21. The observed change in the incidence of new HIV infections (recorded as a percentage of the cohort sample) in a group of male homosexuals in Amsterdam over the time period 1980–1987 (data from van Griesven *et al.* 1989).

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this period, rather than as single-step change. In the absence of more detailed information we attempt to mimic a more continuous pattern of change by introducing two separate step changes in the mean rate of sexual partner change in the sexually active population and among new recruits to this population. These two changes (under the assumptions of scheme 3) are first from 8.7 to 5.0 per year and second from 5.0 to 3.35 per year. The simulated trajectories of the incidence of AIDS through time, for different times for the introductions of the two step changes in behaviour are recorded in figure 22. On the basis of the evidence recorded in the study of Evans *et al.* (1989) in London from 1984–1987, introducing the first step change in 1986 would appear to most closely mirror what actually happens in the U.K. Admittedly on the basis of limited information, we therefore tentatively suggest that the likely pattern of the epidemic in the male homosexual population in England and Wales may be similar to the lower

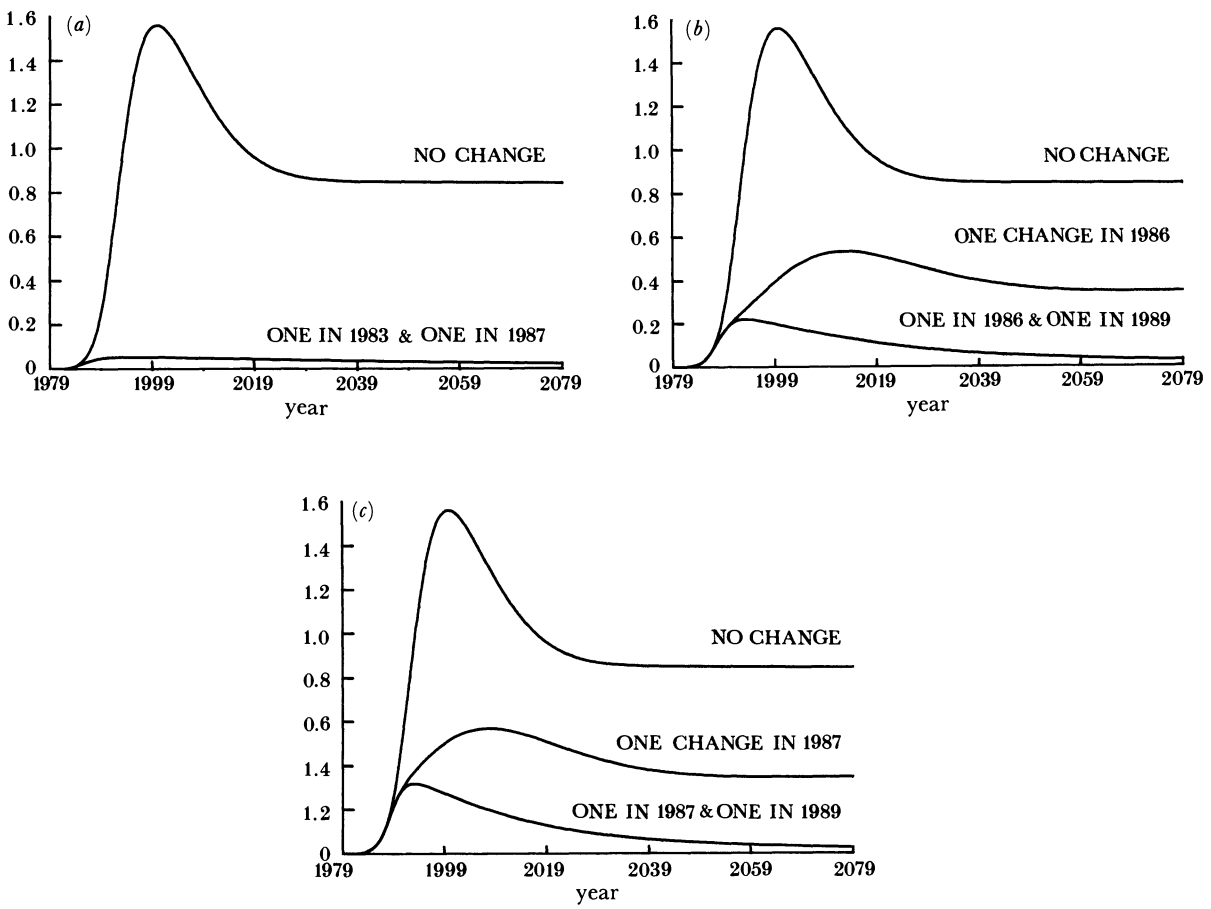

FIGURE 22. The influence of two consecutive reductions in sexual partner change rates on the quarter yearly incidence of AIDS. The parameter set used in the simulations was as defined for run number 2 in table 5 and the assumption concerning how the changes occurred and who they influence was as defined by scheme 3. In the different graphs, the two step changes were introduced at different time points. In (*a*) the changes occurred in 1983 and 1987, in (*b*) they occurred in 1986 and 1989, and in (*c*) they occurred in 1987 and 1989. The change introduced at the first time point was a reduction in the mean rate of partner change from 8.7 to 5.0 per year and that induced at the second time point was from 5.0 to 3.3 per year. For the purpose of comparison simulations under a one step change (from a mean rate of 8.7 per year to a mean of 5.0 per year) and no change at all (a mean of 8.7 per year throughout) are also recorded in graphs (*a*)–(*c*).

trajectories recorded in figure 22*b*, *c*. In these simulations, the incidence of AIDS peaks at around 200–300 new cases per quarter year in 1989–1990 and thereafter slowly declines over a period of many decades.

5. Discussion

Our numerical studies of the behaviour of a simple model of the transmission of HIV in a male homosexual population well illustrate the dangers inherent in making predictions about the future course of the AIDS epidemic in the U.K. Even simple models, based on the assumption of proportionate mixing, contain many parameters associated with the description of the incubation and infectious periods, and the distribution of rates of sexual partner change. Estimates of the values of these parameters are difficult to derive on the basis of published epidemiological research so far. Far too little attention has been devoted to the quantitative measurement and description of the parameters and processes that determine the overall pattern of the epidemic. This obviously needs to be rectified in future epidemiological research. However, even with the best intentions in mind many practical and ethical problems surround the measurement of many of the key parameters. As such, a marked improvement in the current state of affairs is unlikely to occur in the next few years. We are therefore left in the difficult position of trying to provide scientific information about future trends for health-care planning on a rather flimsy web of empirical data. On the basis of past trends in the rise in incidence of AIDS through time, plus limited data on the mean level of sexual activity within homosexual communities over the period 1979–1986, it seems likely that the average transmission probability $\bar{\beta}$ is of the order of 0.01. However, the relative durations (T_is) and intensities (β_is) of the two phases of infectiousness can have a marked impact on the course of epidemic even within the constraint of an average rate (β) of the magnitude of 0.01. If we assume that the first and last phases of infectiousness have durations of 1 and 3 years respectively (with a $\bar{\beta}$ of 0.01) then the model can provide a reasonable fit to observed trends even in the absence of any assumption concerning changes in sexual behaviour. A comparison of observation with model prediction is presented in figure 23 with the assumption that the yearly immigration rate of newly infected persons from Africa and the U.S.A. was of the order of 20 per year from 1979 to 1985 (we believe this to be a reasonable assumption on the basis of published data on risk factors associated with diagnosed cases of AIDS in the U.K. over this period). Aside from the uncertainties associated with the magnitude of the T_is and the β_is, future trends will be greatly influenced by the average incubation period of the disease. In the absence of nationwide serological data for the general population we are unable at present to check on our assumption of an eight-year average incubation period, by relating reported cases to the number of infected persons in the male homosexual population. Our estimate of the average period may well increase as data accumulates. Resolution of this problem would be greatly facilitated by good serological data derived from large-scale surveys (anonymous screening without consent).

Changes in sexual behaviour throughout the course of the epidemic further complicate the issue of predicting future trends. Changes have undoubtedly occurred but data are not available on their type and magnitude. The numerical studies of model behaviour under differing assumptions concerning the form and type of such changes clearly illustrate that a detailed knowledge of how such changes have taken place, and who has changed behaviour,

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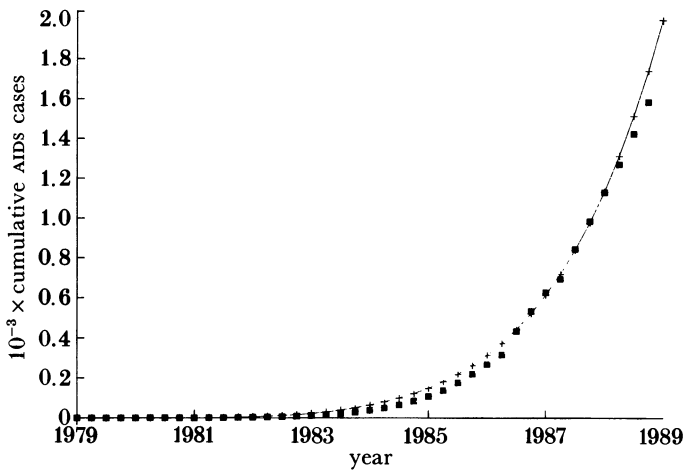

FIGURE 23. A comparison of the observed trend in reported AIDS cases in the U.K. among male homosexuals (recorded as cumulative cases) and the predictions of the model. (■) Data, (+) model. The epidemic was initiated in the model simulation in 1979 and the immigration rate of infected persons from outside the U.K. (i.e. from the U.S.A. or Africa) was set at 20 per year over the period 1979–1985. The parameter set employed was as defined for simulation run number 2 in table 5. Sexual behaviour was unchanged in the simulation in the period 1979–1989.

is essential for making projections into the future. At present the importance of recording summary statistics of the full distribution of rates of sexual partner change per unit of time is not widely appreciated by those concerned with research on sexual behaviour. For example a recent study by Fay *et al.* (1989), on patterns of sexual behaviour among male homosexual populations in the U.S.A. in 1979 and 1988 is excellent in the attention given to sampling method and data reliability. However, information on distributions of rates of sexual partner change per lifetime of same-gender sexual activity (clearly stratified by age and other factors) is not presented in a form that either records, or allows the calculation of summary statistics such as the mean or variance. This problem requires careful attention in future studies on sexual behaviour if our understanding of the transmission dynamics of HIV is to improve. The major issue illustrated by the numerical studies of simulated changes in sexual behaviour concerns the benefits to be gained from the induction of such changes early in the timecourse of the epidemic. This point is most clearly illustrated in figure 22. A simple comparison of the top and bottom trajectories of predicted cases of AIDS in graphs (*a*)–(*c*) reveals the enormous benefit that accrues from a reduction in the mean rate of sexual partner change (and an associated reduction in the variance) from around 8.7 to 3.3 per year in two steps early on in the protected timecourse. The induced reduction in the mean is of the order of 60% (from its pristine value) and yet this results in more than a 90% reduction in the cumulative number of AIDS cases over the 100-year timespan of the simulated epidemic. The moral is clear: resources used in education and publicity early on in the epidemic have a much greater impact in the long term, than resources used at a later stage. A second point of practical importance illustrated by the simulations concerns the question of who to target education at. The numerical studies reveal that the greatest immediate impact on the rate at which new cases of infection arise is achieved if reductions in the rate of sexual partner change occur in the infected subpopulation. In other words, education that aims to change behaviour must be directed not

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only at those who have not as yet joined the sexually active individuals who are still uninfected, but also, and most importantly, at those who are already infected (see figure 19).

One of the major objectives of the present study was to attempt to provide some general insights (both qualitative and quantitative) into the likely pattern of the epidemic in male homosexuals in the U.K. over the coming decades. We have stressed the difficulties inherent in attempting such projections, given the paucity of knowledge about key epidemiological parameters and processes. However, given the urgency of providing some guidelines, even if based on limited data, we draw attention to the projections recorded in figure 22. Under the crude assumption of a reduction in the mean rate of sexual partner change from 8.7 per year in 1985 to 5.0 per year in 1986 and to 3.35 per year in 1989 model projections suggest that the incidence of AIDS may plateau at around 200–300 cases per quarter year in 1989/90 and thereafter slowly decline over a period of many decades. The number of infected male homosexuals (HIV seropositive) in the U.K. at the period of peak incidence of disease is predicted to be somewhere between 10000–15000. If the average incubation period is significantly longer than eight years then the above figures will underestimate the true picture and we may expect a longer linear phase of the epidemic (reported cases of AIDS) over the coming five years.

The model employed in our numerical studies can be subject to various improvements and refinements. The major areas that require attention, aside from better parameter estimates, concern the assumption of proportionate mixing in sexual contacts and the need to introduce age structure to take account of changes in sexual activity with age. The question of 'who mixes with whom' with respect to individuals in different sexual activity classes is an important one. A number of recent studies have attempted to assess the impacts of different assumptions ranging from the extreme of virtually all contacts occurring within a given sexual activity class to the baseline assumption of proportionate mixing (Koopman *et al.* 1988; Hyman & Stanley 1988; May & Anderson 1989). Such studies are unfortunately rather abstract in character at the moment because of the absence of any data on the degrees of connectance within and between sexual activity classes. There is an urgent need for information in this area. The issue of age dependency is of obvious importance and this could be relatively easily incorporated within the structures of existing models. Limited data exist on patterns of change with age in the rate of acquiring new sexual partners (see Fay *et al.* 1989; Anderson 1988*b*). Fully age-structured models that allow varying degrees of mixing between and within different sexual activity classes will of course contain many more parameters for which estimates are required. An additional problem is the scale and cost of the numerical problems surrounding the investigation and analysis of the behaviours of such models.

In conclusion, we have restricted our attention to the spread of HIV among male homosexuals and tentatively suggested that changes in sexual behaviour may have resulted in the epidemic being close to, or at, its point of peak incidence (with respect to cases of AIDS) at present. This is encouraging but it should not be interpreted as implying that the problem is being conquered in the U.K. First, and foremost, the observed changes in behaviour among male homosexuals must be maintained in order to prevent a second upsurge in the incidence of new infections. Second, the virus is continuing to spread in other high-risk groups such as heterosexuals and intravenous drug users. A recent survey in London, for example, recorded a prevalence of HIV antibody of 1.6% in heterosexuals and 5.7% in intravenous drug users (Public Health

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Laboratory Service Working Group 1989). A different study, also in London, recorded a rise in seroprevalence of HIV antibody in heterosexuals attending sexually transmitted disease clinics from 0.5% in January 1986 to 1.0% in 1987 (Loveday *et al.* 1989). These figures give cause for concern. Whether or not there will be a widely disseminated epidemic among heterosexuals in the U.K. remains uncertain. However, as we have noted elsewhere, if the case reproductive rate among heterosexuals is just above unity, the doubling time of the epidemic will be of the order of eight or more years (Anderson & May 1988). As such, in the first decade of its spread, changes in the general population from a fraction of 1% infected to some large fraction of 1% infected, will be extremely difficult to detect. Continued vigilance and increased efforts to improve epidemiological knowledge are clearly required under these circumstances.

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Appendix A

At a non-trivial steady state we have:

$$0 = \Lambda(s) - (\lambda^* s + \mu)\, x^*(s), \tag{A 1}$$

$$\frac{\partial y^*(u,s)}{\partial u} = -(v(u)+\mu)\, y^*(u,s), \tag{A 2}$$

$$y^*(0,s) = \lambda^* s x^*(s), \tag{A 3}$$

with

$$\lambda^* = \frac{\int_0^\infty s' \int_0^\infty \beta(u)\, y^*(u,s)\, \mathrm{d}u\, \mathrm{d}s'}{s'\left[x^*(s) + \int_0^\infty y^*(u,s)\, \mathrm{d}u\right]\mathrm{d}s'}. \tag{A 4}$$

Writing

$$Y^*(s) = \int_0^\infty y^*(u,s)\, \mathrm{d}u, \tag{A 5}$$

then (A 2), (A 3) become

$$\begin{aligned} 0 &= \lambda^* s x^*(s) - \int_0^\infty v(u)\, y^*(u,s)\, \mathrm{d}u - \mu Y^*(s), \\ &= \lambda^* s x^*(s)\,(1-Q) - \mu Y^*(s) \end{aligned} \tag{A 6}$$

(Blythe & Anderson 1988*a*, *c*) where

$$Q = \int_0^\infty f(u)\, \mathrm{e}^{-\mu u}\, \mathrm{d}u, \tag{A 7}$$

and

$$f(u) = v(u) \exp\left\{-\int_0^u v(u'\, \mathrm{d}u')\right\}, \tag{A 8}$$

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in the p.d.f. of the incubation period. Writing

$$B(s) = \lambda^* s x^*(s)\, L, \tag{A 9}$$

$$L = \int_0^\infty S(u)\, \mathrm{e}^{-\mu u}\, \mathrm{d}u, \tag{A 10}$$

and

$$S(u) = \sum_{j=1}^{N} (\beta_j/\sigma_j) \sum_{i=1}^{j} b_{j,i}\, \mathrm{e}^{-\sigma_j u}, \tag{A 11}$$

for $f(u)$ described by a series of N distinct exponential distributions with decay parameters $\{\sigma_j\}$, for each of which there is a fixed level of infectiousness $\{\beta_j\}$, with

$$b_{ji} = \prod_{k=1}^{j} \sigma_k \prod_{\substack{k=1 \\ k \neq i}}^{i} (\sigma_k - \sigma_j) \tag{A 12}$$

(details in Blythe & Anderson 1988*b*), (A 4) may now be written

$$\lambda^* = \frac{\int_0^\infty s' \beta(s')\, \mathrm{d}s'}{\int_0^\infty s' [x(s') + Y^*(s')]\, \mathrm{d}s'}, \tag{A 13}$$

which, using (A 6), becomes

$$\int_0^\infty s^* x^*(s)\, \mathrm{d}s + \int_0^\infty s^2 x^*(s)\, \mathrm{d}s \left[\frac{(1-Q)}{\mu} \lambda^* - L \right] = 0. \tag{A 14}$$

Hence defining

$$\hat{s} = \int_0^\infty s^2 x^*(s)\, \mathrm{d}s \Big/ \int_0^\infty s x^*(s)\, \mathrm{d}s \tag{A 15}$$

(see Blythe & Anderson 1988*c*), we have

$$\lambda^* = \frac{\mu}{1-Q} (L - 1/\hat{s}), \tag{A 16}$$

which provides condition

$$\rho = L\hat{s} > 1, \tag{A 17}$$

for endemic persistence of the infection.

For the multi-stage infection model (Blythe & Anderson 1988*b*), we have

$$L = \sum_{j=1}^{N} \frac{\beta}{\sigma_j} \sum_{i=1}^{j} \frac{b_{ji}}{\sigma_j + \mu}. \tag{A 18}$$

In the model and in this paper, $\beta_1 > 0$, $\beta_2 = 0$, $\beta_3 > 0$, $\sigma_1 \neq \sigma_2 \neq \sigma_3$ so (A 18) becomes,

$$L = \frac{\beta_1}{\sigma_1 + \mu} + \frac{\beta_3}{k_2} \frac{\sigma_1 \sigma_2 k_1}{k_3}, \tag{A 19}$$

where

$$k_1 = (\sigma_2 + \mu)(\sigma_3 + \mu)(\sigma_3 - \sigma_2) - (\sigma_1 + \mu)(\sigma_3 + \mu) + (\sigma_3 - \sigma_1) + (\sigma_1 + \mu)(\sigma_2 + \mu)(\sigma_2 - \sigma_1), \tag{A 20}$$

$$k_2 = (\sigma_1 + \mu)(\sigma_2 + \mu)(\sigma_3 + \mu), \tag{A 21}$$

$$k_3 = (\sigma_2 - \sigma_1)(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2). \tag{A 22}$$

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Following Blythe & Anderson (1988*c*), we obtain values for λ^* and $\hat{s}$ by solving the equation

$$\int_0^\infty \frac{s\Lambda(s)}{s\lambda^*+\mu}\left\{1+s\left[\frac{(1-Q)}{\mu}\lambda^*-L\right]\right\}\mathrm{d}s=0, \tag{A 23}$$

for λ^*, and calculating

$$\hat{s}=\mu/[\mu L-(1-Q)\,\lambda^*]. \tag{A 24}$$

Defining X_i^* as the steady-state population size of the ith susceptible class, and

$$x_i^*=\int_{s_i}^{s_{i+1}}\frac{\Lambda(s)}{s\lambda^*+\mu}\mathrm{d}s, \tag{A 25}$$

as the steady-state value for equivalent range of s in the continuum model, we may follow Blythe & Anderson (1988*c*) and define:

$$s_i=\frac{1}{\lambda^*}(\Lambda_i/x_i^*-\mu). \tag{A 26}$$

Then if the jth activity class has activity level c_j, and the rest have levels $c_i=s_i (i\neq j)$, then $c_j=s_j'$, any solution of

$$s'=\tfrac{1}{2}[\hat{s}+\lambda^*K\pm[(\hat{s}+\lambda^*K)^2+4\mu K]^{\frac{1}{2}}, \tag{A 27}$$

where

$$K=-\sum_{\substack{i=1\\ i\neq j}}^{N} s_i(s_i-s)\,x_i^*. \tag{A 28}$$

Subject to the constrain $s_j\leqslant s_j'<s_{j+1}$.

The choice of the 'balancing class' j is arbitrary, but for $N=6$ Blythe & Anderson (1988*c*) found $j=5$ was an appropriate choice.

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